Discrete Mathematics

(Comp 4190.101)

Final Exam: Fall, 2001

Problem	Score	Problem	Score
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	
		Total	

Name: ____________________

ID No: ____________________

Phone: ____________________

E-mail: ____________________

1. (5 points) Determine whether the following directed graph has an Euler circuit. Explain why.

2. (5 points) Determine whether the following directed graph has an Euler path. Explain why.

3. (10 points) Solve the traveling salesman problem for the following graph.

4. (5 points) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?

5. (10 points) The mathematics department has six committees that meet once a month. How many different meeting times must be used to ensure that no one is scheduled to be at two meetings at the same time if the committees are

(a) $C_1 = \{\text{Arlinghaus}, \text{Brand}, \text{Zaslavsky}\}$,

(b) $C_2 = \{\text{Brand}, \text{Lee}, \text{Rosen}\}$,

(c) $C_3 = \{\text{Arlinghaus}, \text{Rosen}, \text{Zaslavsky}\}$,

(d) $C_4 = \{\text{Lee}, \text{Rosen}, \text{Zaslavsky}\}$,

(e) $C_5 = \{\text{Arlinghaus}, \text{Brand}\}$,

(f) $C_6 = \{\text{Brand}, \text{Rosen}, \text{Zaslavsky}\}$.

6. (a) (2 points) What is the value of the prefix expression: `* / 9 3 + * 2 4 - 7 6` ?

   (b) (2 points) What is the value of the postfix expression: `9 3 / 5 + 7 2 - *` ?

7. (15 points) How many weightings of a balance scale are needed to find a counterfeit coin among eight coins if the counterfeit is either heavier or lighter than the others? Describe your algorithm.

8. (7 points) Use Prim's algorithm to find a minimum spanning tree for the following weighted graph.

9. (7 points) Use Kruskal's algorithm to find a minimum spanning tree for the following weighted graph.

10. (a) (3 points) How many squares does a Karnaugh map in six variables have?

(b) (4 points) How many squares are adjacent to a given square in a Karnaugh map in six variables?

11. (10 points) How many different Boolean functions $F(x, y, z)$ are there so that $F(\bar{x}, y, z) = F(x, \bar{y}, z) = F(x, y, \bar{z})$ for all values of the Boolean variables x, y, z?

12. (15 points) Use the Quine-McCluskey method to simply the following sum-of-product expansion:

$$wxyz + wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}xyz + wx\bar{y}\bar{z} + w\bar{x}\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z}$$