Engineering Mathematics I
(Comp 400.001)

Final Exam: June 10, 2000

Problem	Score	Problem	Score
1		6	
2		7	
3		8	
4		9	
5		10	
		Total	

Name: ____________________

ID No: ____________________

1. (10 points) Solve the following initial value problem

$$y''' + 3y'' + 3y' + y = 30e^{-x}, \quad y(0) = 3,\ y'(0) = -3,\ y''(0) = -47.$$

2. (10 points) Find a general solution for the following equation

$$x^3y''' - 3x^2y'' + 6xy' - 6y = x^4 \ln x \quad (x > 0).$$

3. (15 points) Find a general solution for the following equation

$$\begin{aligned} y_1' &= 2y_1 + 3y_2 - 2e^{-t}, \\ y_2' &= 13y_1 - 6y_2. \end{aligned}$$

4. (15 points) Solve the mixed boundary value problem for the Laplace equation $\nabla^2 u = 0$ with the boundary data and grid as given in Figure 426.

5. (10 points) Find the Fourier transform of the following function

$$f(x) = \begin{cases} |x|, & \text{if } -1 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

6. (15 points) Find the Fourier series of the following periodic function:

$$f(x+4) = f(x) = \begin{cases} 0, & -2 \leq x \leq 1; \\ x, & -1 < x < 1; \\ 0, & 1 \leq x < 2. \end{cases}$$

7. (20 points) Solve the following initial value problem:

$$y'' + \omega^2 y = f(t), \quad y(0) = 0, \; y'(0) = 0,$$

where

$$f(t + 2\pi) = f(t) = \begin{cases} 1, & 0 < t < \pi; \\ 0, & t = 0, \pi, 2\pi; \\ 1, & \pi < t < 2\pi. \end{cases}$$

8. (15 points) Solve the following heat equation:

$$\begin{cases} u_{xx} = 4u_t, \quad 0 < x < 2,\ t > 0 \\ u(0,t) = 0,\ u(2,t) = 0, \quad t > 0 \\ u(x,0) = 2\sin(\pi x/2) - \sin \pi x + 4\sin 2\pi x, \quad 0 \le x \le 2. \end{cases}$$

9. (20 points) Find the solution $u(x, y)$ of Laplace's equation in the rectangle $0 < x < a$ and $0 < y < b$, that satisfies the boundary conditions:

$$\begin{cases} u_x(0, y) = 0, \quad u_x(a, y) = 0, & 0 < y < b, \\ u(x, 0) = 0, \quad u(x, b) = f(x), & 0 \le x \le a. \end{cases}$$

10. (20 points) Consider the heat equation for finding the temperature $w(x,t)$ in a semi-infinite laterally insulated bar extending from $x=0$ along the x-axis to ∞, assuming that the initial temperature is 0, $w(x,t) \to 0$ as $x \to \infty$ for every fixed $t \geq 0$, and $w(0,t) = f(t)$. Show that the Laplace transform leads to

$$sW(x,s) = c^2 \frac{\partial^2 W}{\partial x^2} \quad \text{and} \quad W(x,s) = F(s) e^{\sqrt{s}x/c},$$

where $W(x,s) = \mathcal{L}[w(x,t)]$ and $F(s) = \mathcal{L}[f(t)]$.