Engineering Mathematics I
(Comp 400.001)

Final Exam: June 19, 1999

Problem	Score	Problem	Score
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8		Total	

Name: ______________________

ID No: ______________________

1. (10 points) Solve the following initial value problem

$$y' = y + y^2, \quad y(0) = 1.$$

2. (10 points) Find a general solution for the following equation

$$x^2 y'' - 4xy' + 6y = 0.$$

3. (10 points) Solve the following initial value problem:

$$\begin{aligned} y_1' &= y_1 + y_2, \quad y_1(0) = 4 \\ y_2' &= 4y_1 + y_2, \quad y_2(0) = 4 \end{aligned}$$

4. (10 points) Find the Fourier series of the periodic function $f(x) = x$ $(-\pi \leq x \leq 0)$, $f(x) = 0$ $(0 \leq x \leq \pi)$, and $f(x) = f(x + 2\pi)$.

5. (10 points) Find the Fourier series of the periodic function of period $p = 2L = 2$, $f(x) = x$ $(0 < x < 1)$, $f(x) = 0$ $(1 < x < 2)$, and $f(x) = f(x+2)$.

6. (10 points) Show that

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n-1}=\frac{\pi}{4}$$

using the Fourier series of the function $f(x) = 1$ $(-\pi/2 < x < \pi/2)$, $f(x) = 0$ $(\pi/2 < x < 3\pi/2)$, and $f(x) = f(x+2\pi)$.

7. (10 points) Using the Fourier sine integral, show the following equivalence:

$$\int_0^\infty \frac{w^3 \sin xw}{w^4 + 4} dw = \frac{\pi}{2} e^{-x} \cos x, \quad \text{if } x > 0.$$

8. (10 points) Find the Fourier transform of the function $f(x) = xe^{-x}$ (if $x > 0$), and $f(x) = 0$ (if $x < 0$).

9. (10 points) Solve the following P.D.E. for $u(x, y)$ by separating variables:

$$xu_{xy} + 2yu = 0.$$

10. (10 points) Describe the general procedure for solving the one-dimensional wave equation.

11. (20 points) Consider the two-dimensional wave equation: $u_{tt} = u_{xx} + u_{yy}$, with a boundary condition: $u = 0$ on the boundary of the membrane for all $t \geq 0$, and initial conditions: $u(x, y, 0) = f(x, y)$ and $u_t(x, y, 0) = g(x, y)$. By applying the method of separation of variables: $u(x, y, t) = H(x)Q(y)G(t)$, show that the given P.D.E. problem reduces to three O.D.E. problems:

$$\begin{aligned} \ddot{G} + \lambda^2 G &= 0, \\ \frac{d^2H}{dx^2} + k^2 H &= 0, \\ \frac{d^2Q}{dy^2} + p^2 Q &= 0. \end{aligned}$$

12. (20 points) Solve the following P.D.E. using the Laplace transformation:

$$\begin{aligned} x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} &= xt, \\ u(x,0) &= 0 \quad \text{if } x \geq 0, \\ u(0,t) &= 0 \quad \text{if } t \geq 0. \end{aligned}$$

13. (20 points) Solve the mixed boundary value problem for the Poisson equation: $\nabla^2 u = 3xy$ in the region and for the boundary conditions shown in Figure 1, using the indicated grid.

14. (20 points) Consider a wave equation $u_{tt} = u_{xx}$ $(0 \leq x \leq 1,\ t \geq 0)$, with initial conditions: $u(x,0) = f(x)$, and $u_t(x,t) = g(x)$, and boundary conditions: $u(0,t) = u(1,t) = 0$, for all $t \geq 0$. Apply the explicit method of numerical solution with $h = k = 0.2$ to the problem, where $0 \leq t \leq 0.4$, $f(x) = x$ (for $0 \leq x \leq 0.2$), $f(x) = 0.25(1-x)$ (for $0.2 \leq x \leq 1$), and $g(x) = x - x^2$ (for $0 \leq x \leq 1$).

15. (20 points) Consider a laterally insulated metal bar of length 1 and satisfying the heat equation $u_t = u_{xx}$. Suppose that the ends of the bar kept at temperature $u(0,t) = u(1,t) = 0$ and the initial temperature in the bar is $f(x) = x$, if $0 \leq x \leq 0.2$, and $f(x) = 0.25(1-x)$, if $0.2 \leq x \leq 1$. Applying the Crank-Nicolson method with $h = 0.2$ and $r = 1$, find the temperature $u(x,t)$ in the bar for $0 \leq t \leq 0.08$.