2

http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

$$\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$
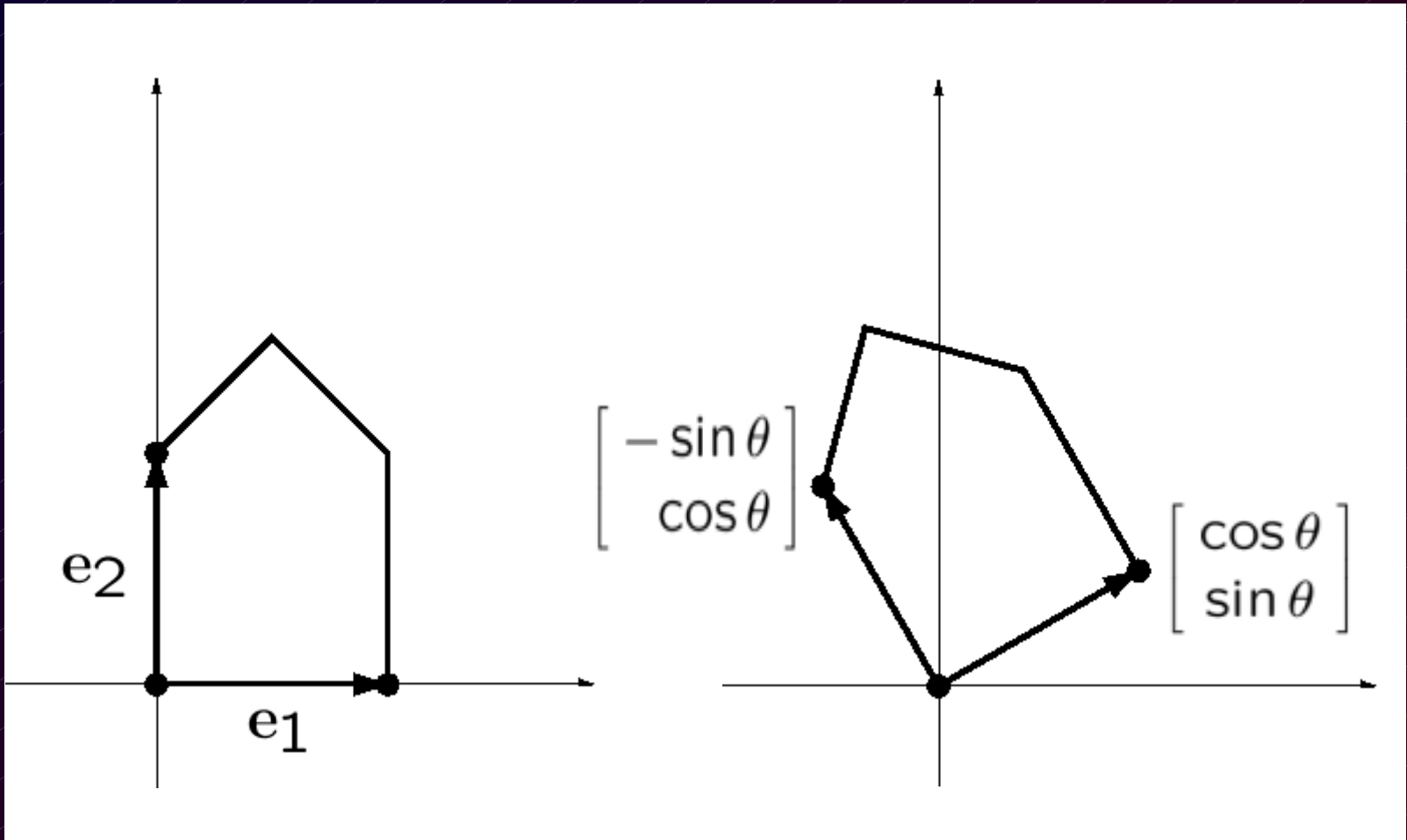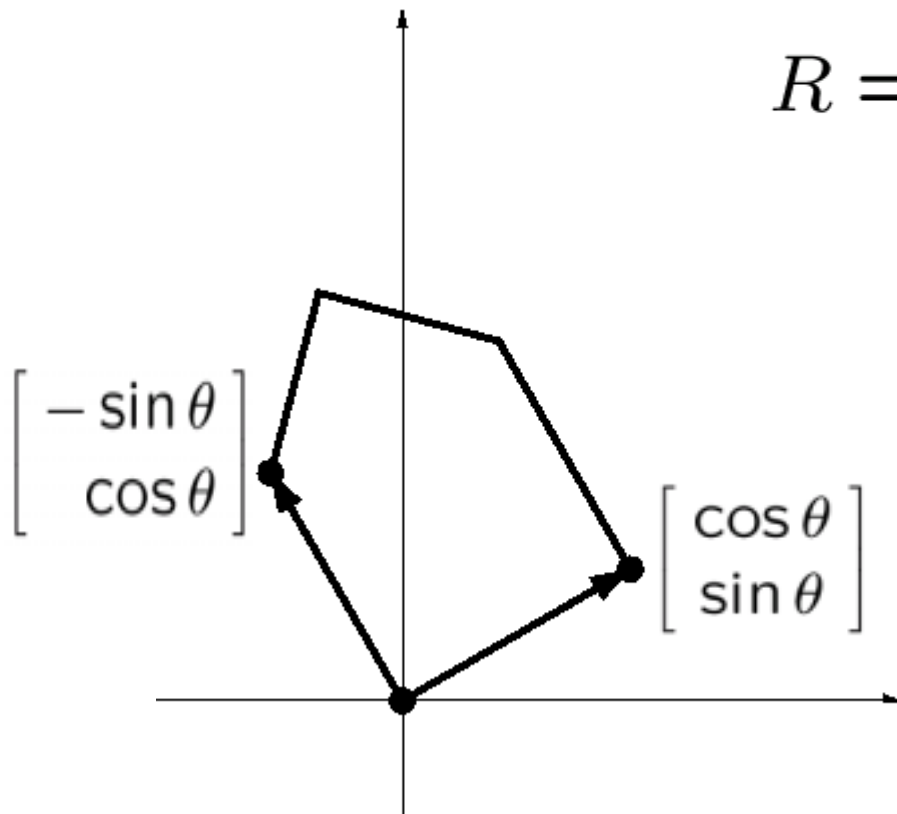
2

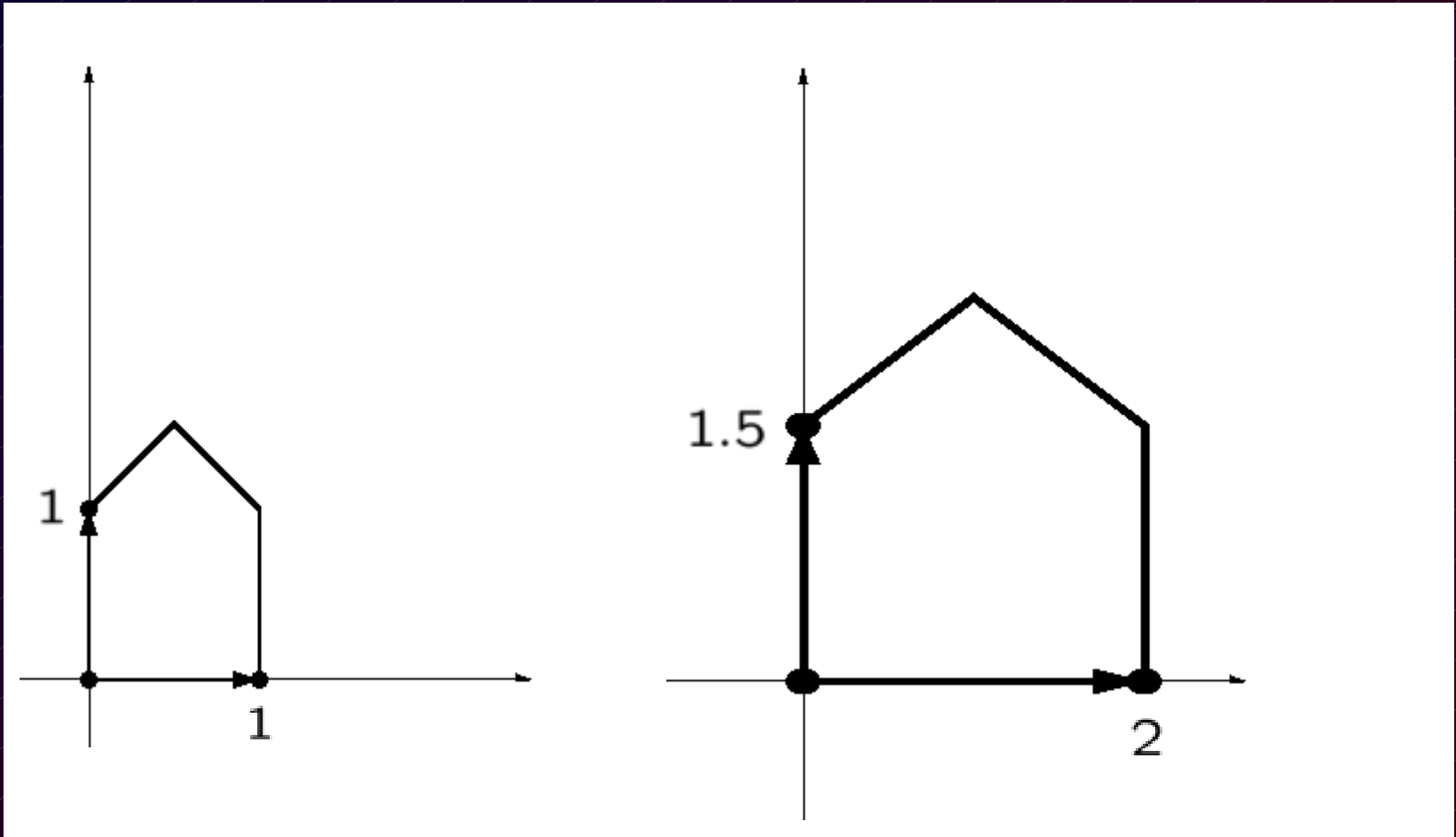$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$
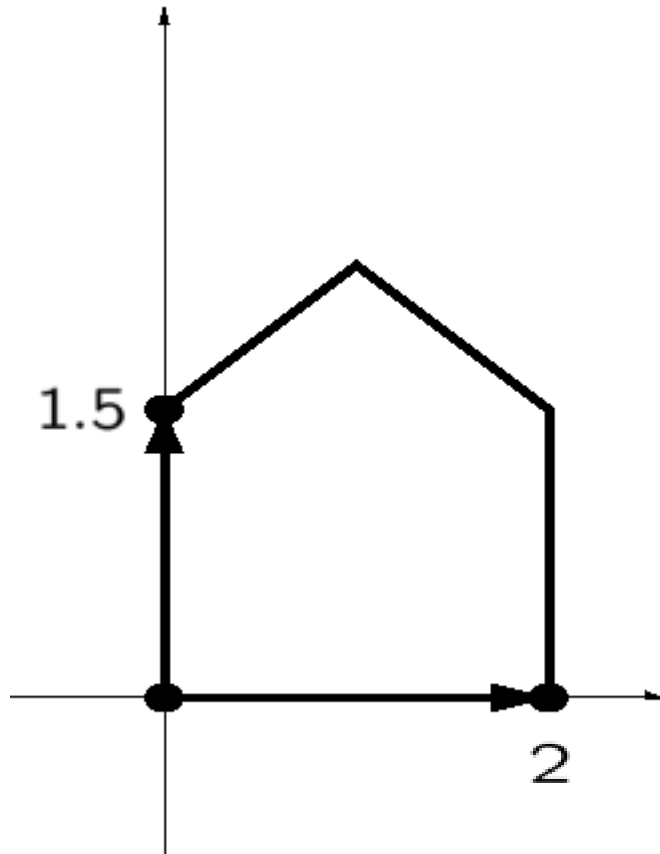
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$
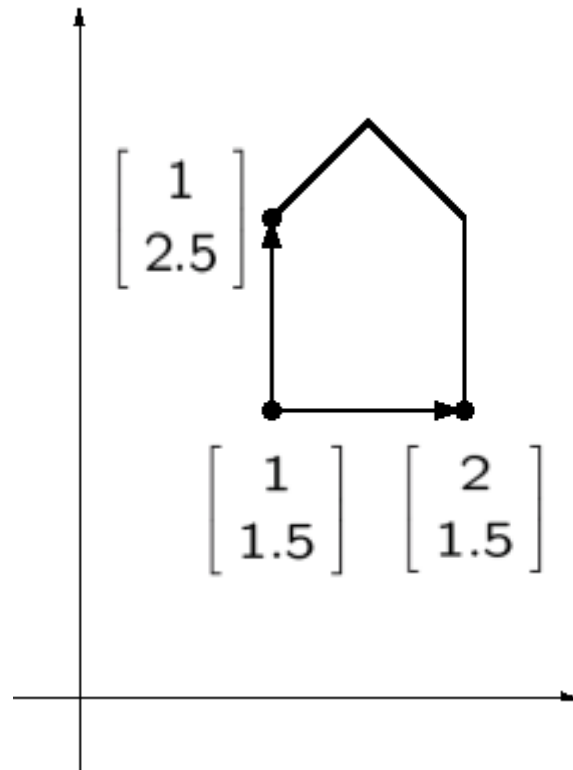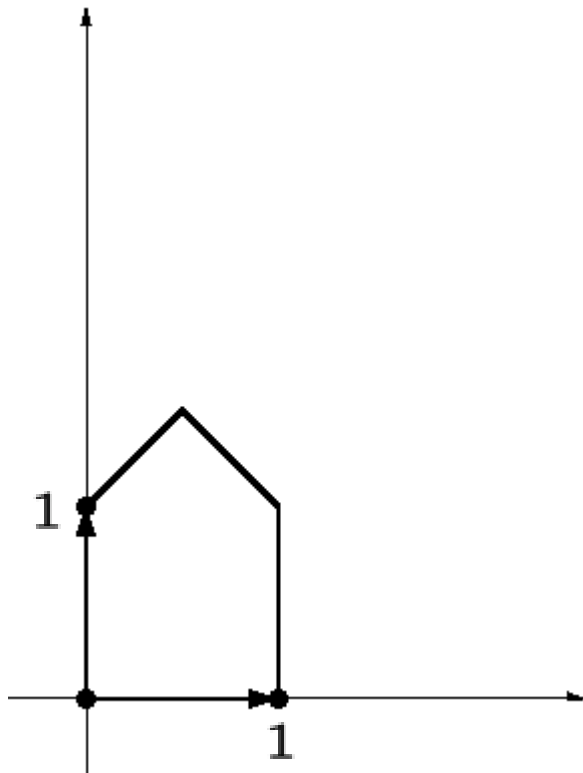
2

2

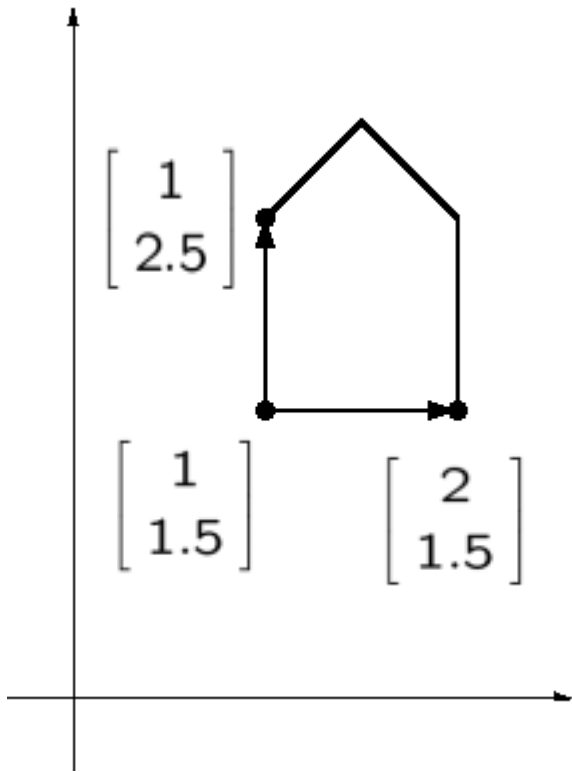$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 1.5y \end{bmatrix}$$

2

2

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} x + 1.0 \\ y + 1.5 \end{bmatrix}$$

2

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$
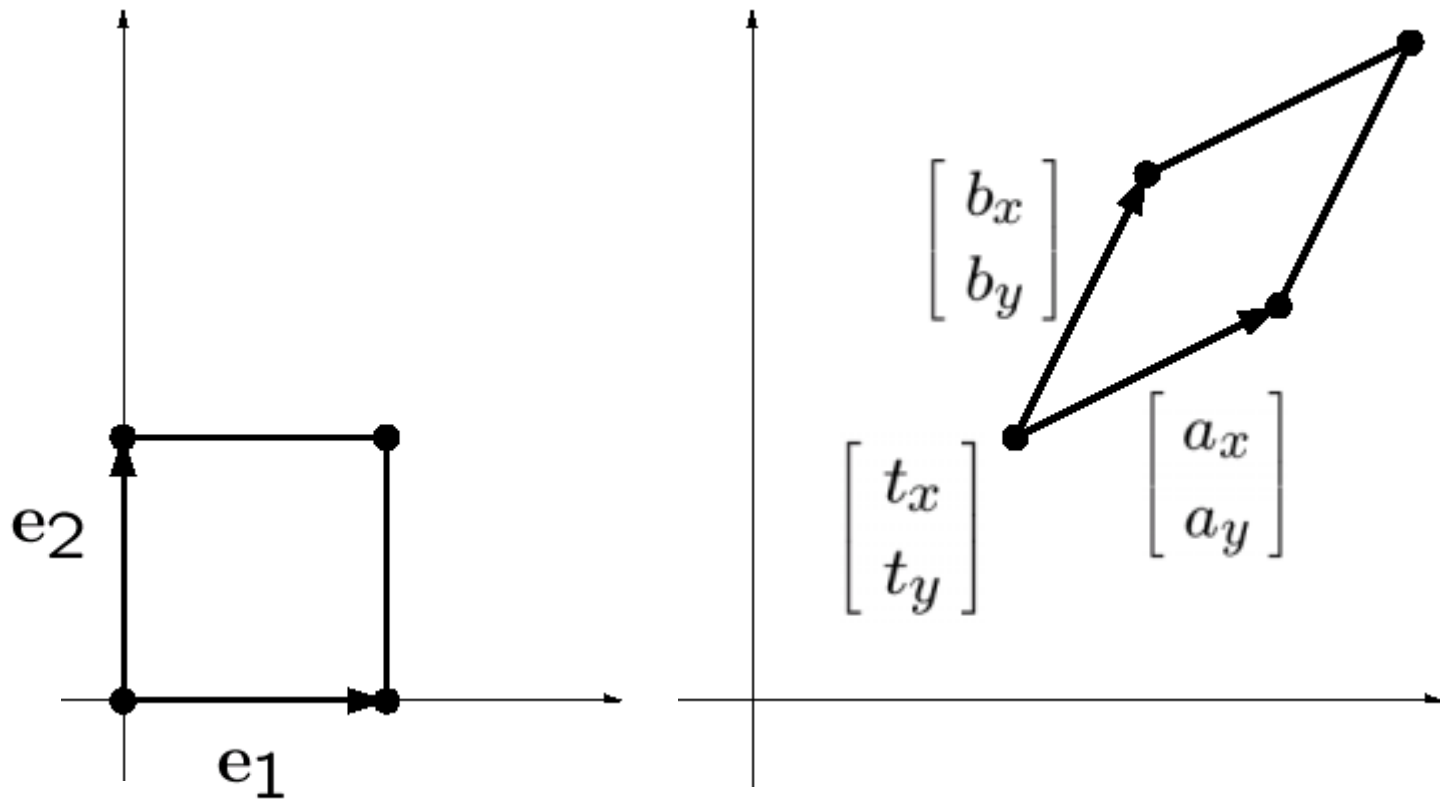
Homogeneous

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} a_x & b_x \\ a_y & b_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

2

$$\begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

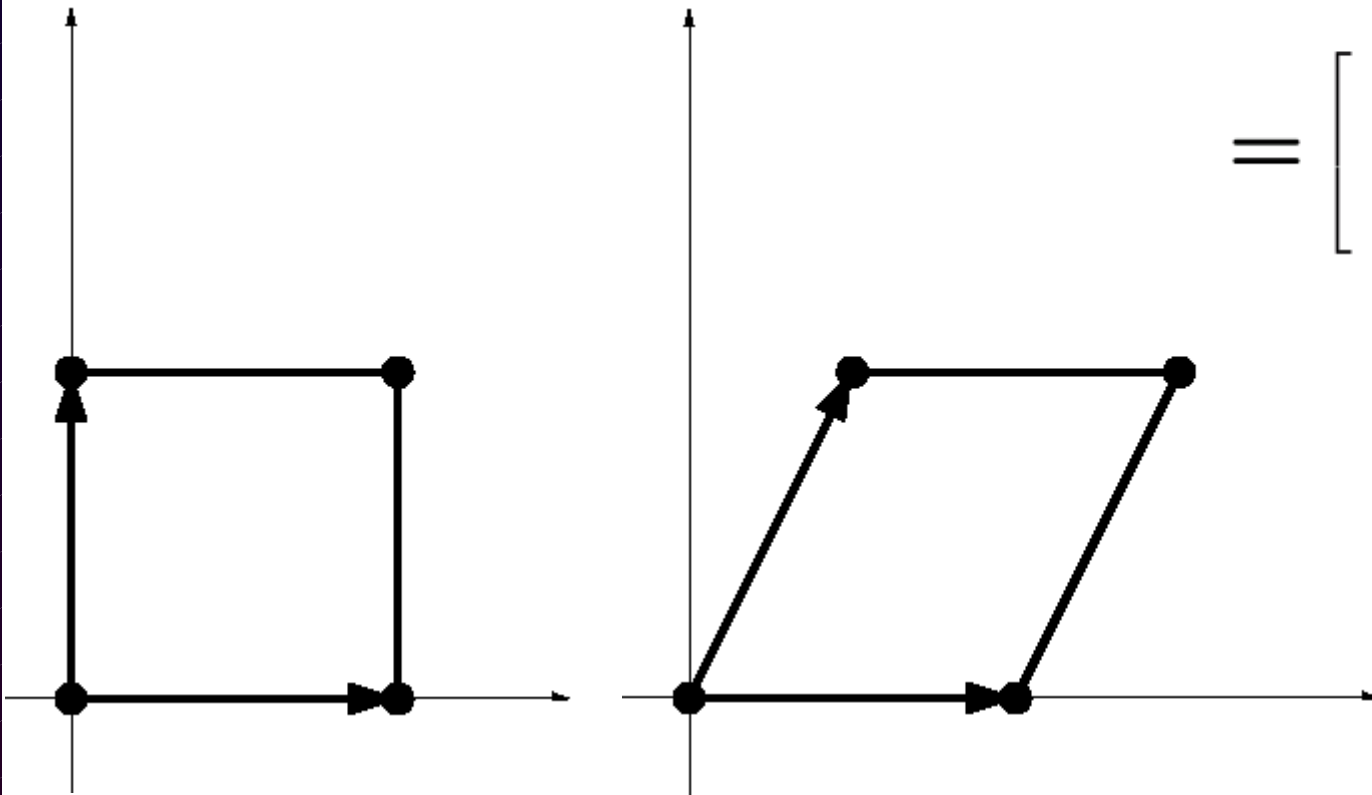$$\begin{bmatrix} b_x \\ b_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2   x-shearing

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
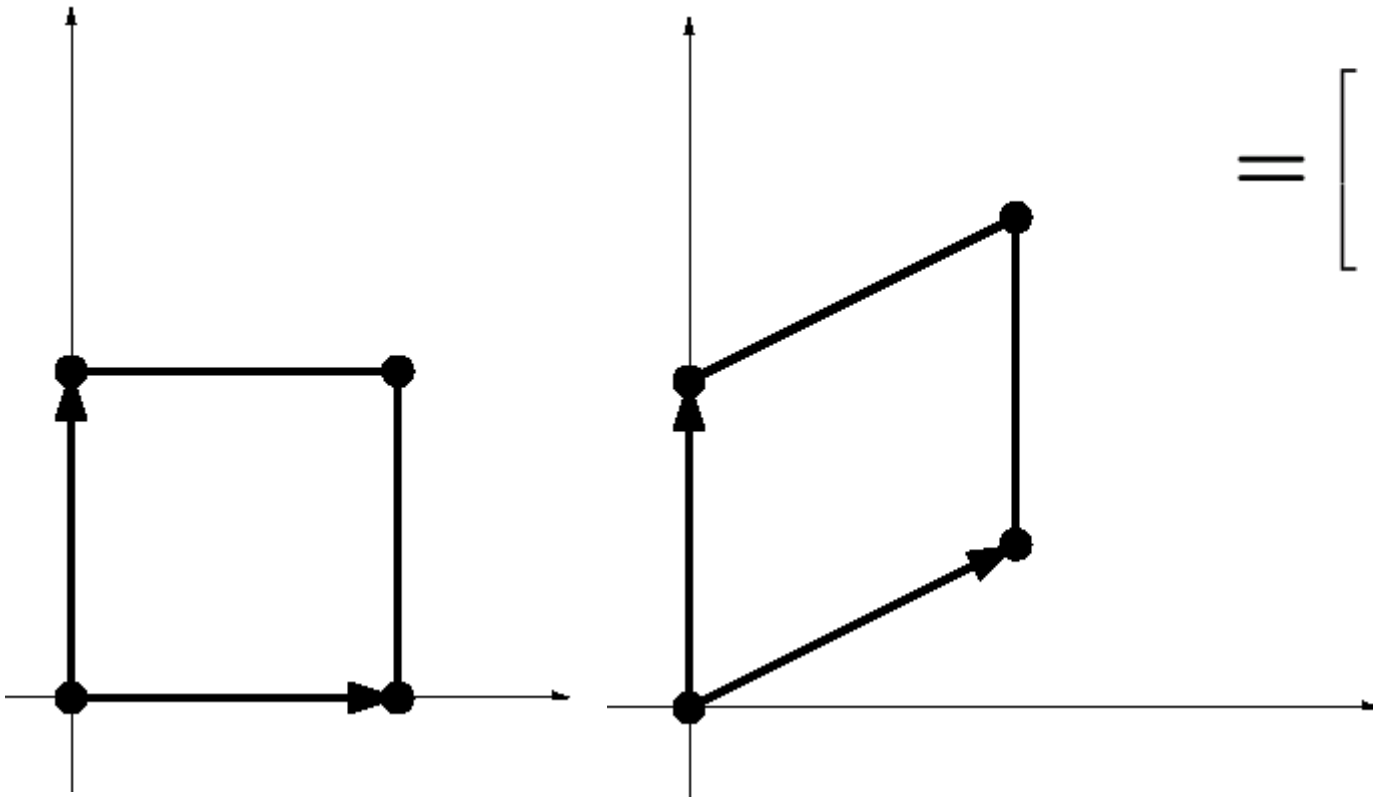
$$= \begin{bmatrix} x + \alpha y \\ y \end{bmatrix}$$
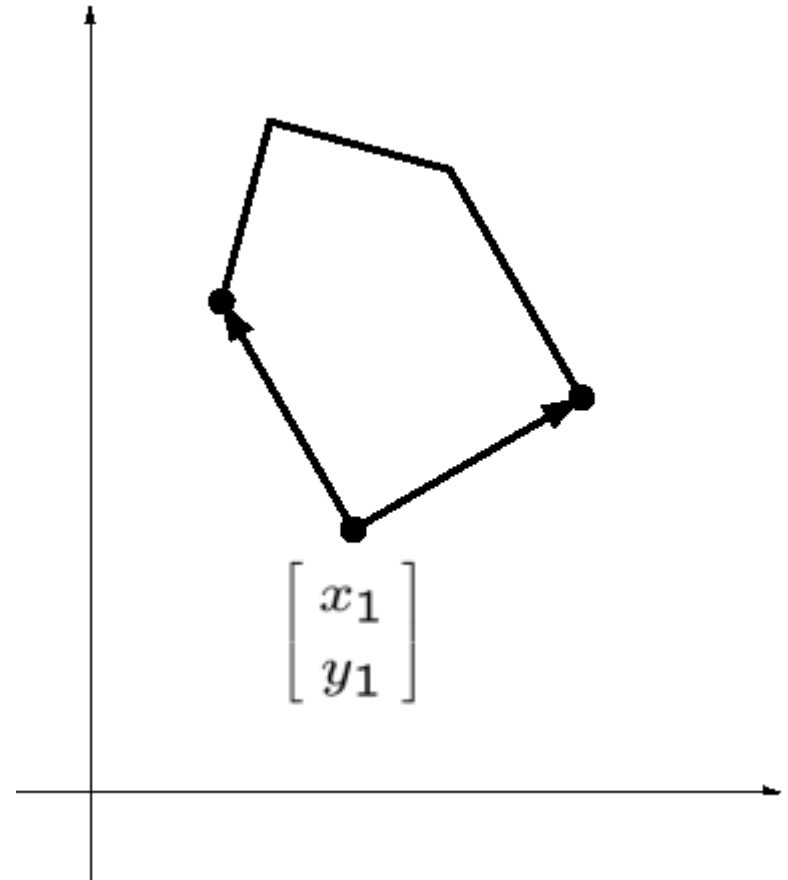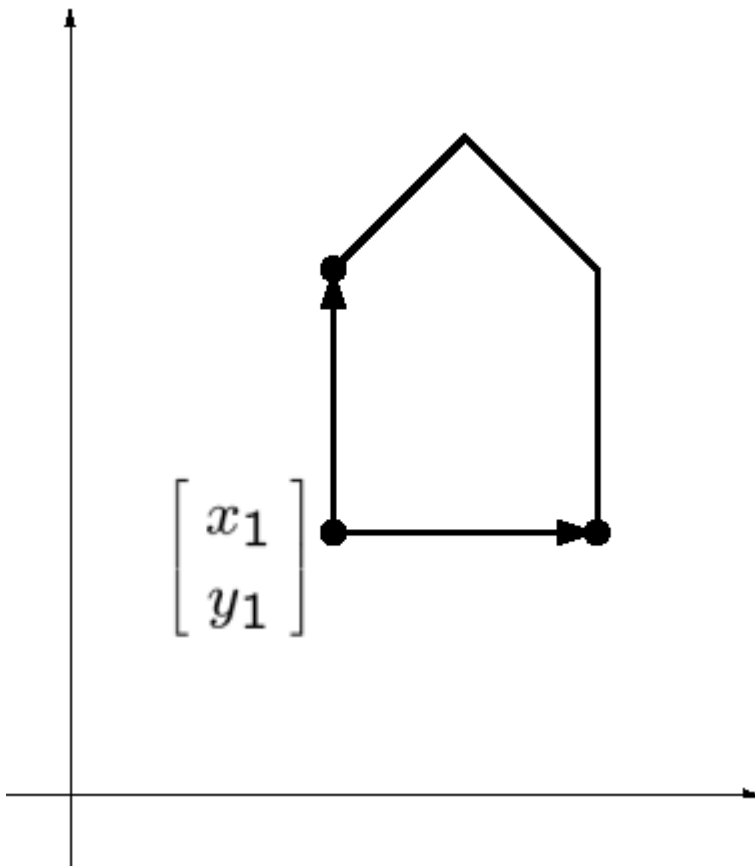
2   y-shearing

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
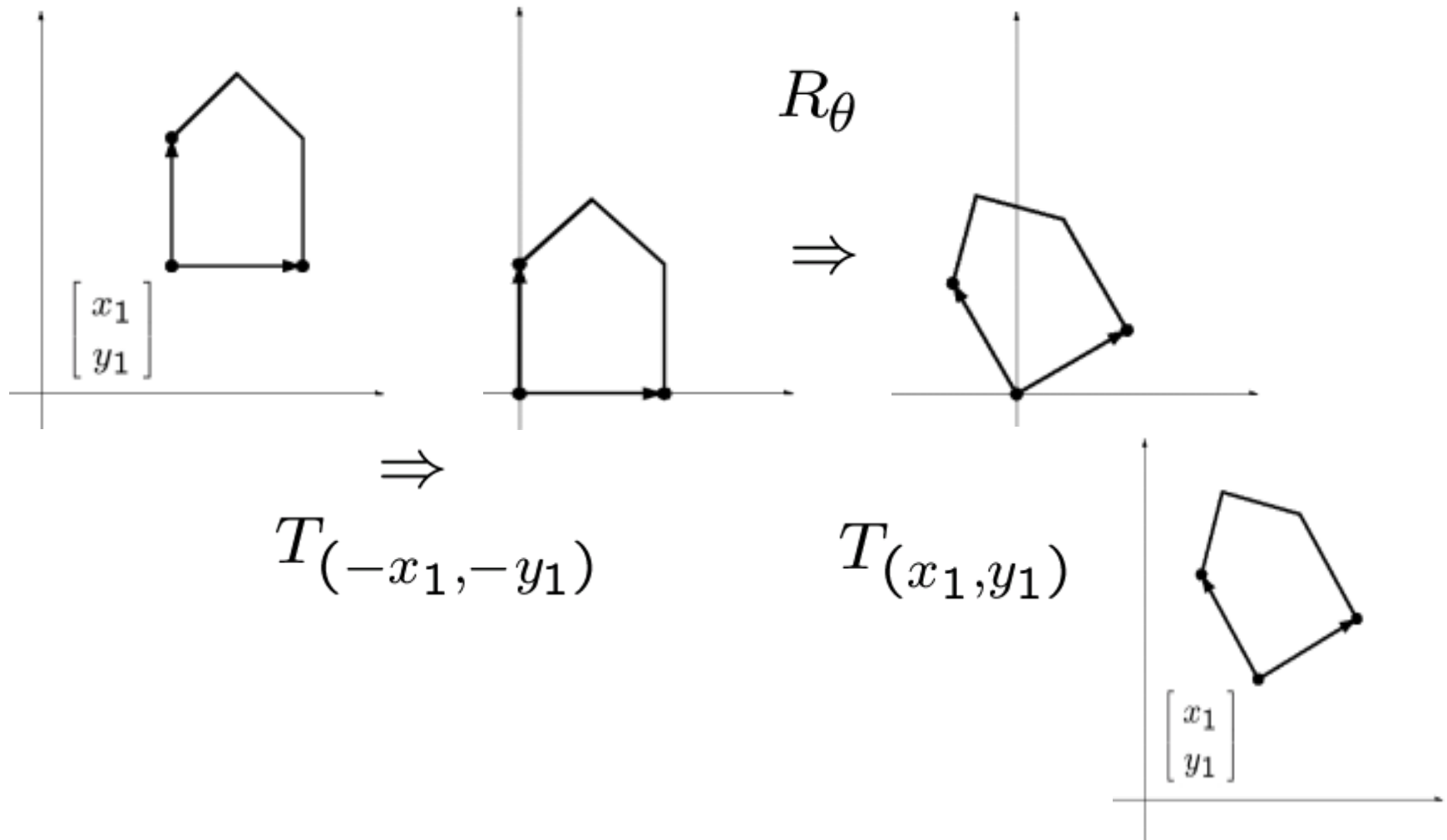
$$= \begin{bmatrix} x \\ \beta x + y \end{bmatrix}$$

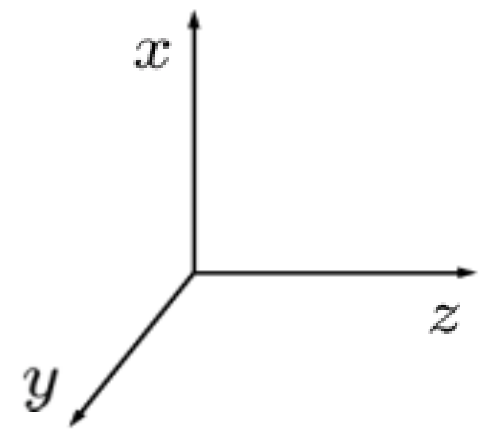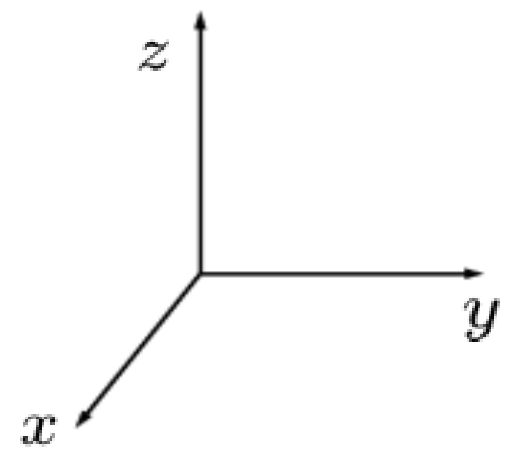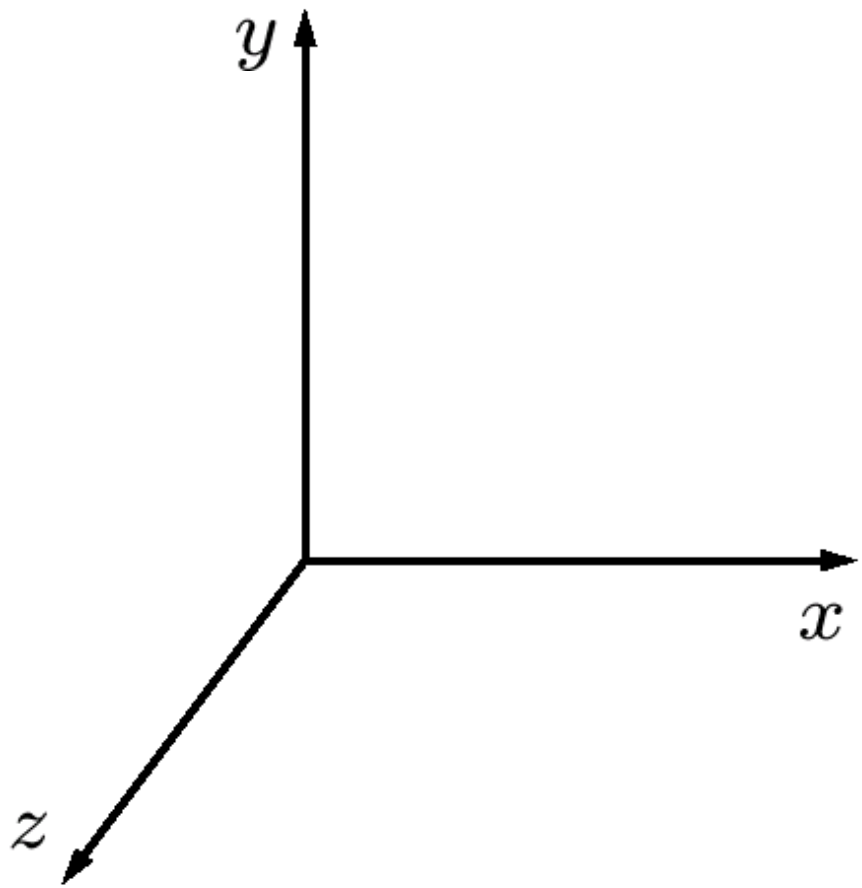$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$R_\theta$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$\Rightarrow$$

$$T_{(-x_1, -y_1)}$$

$$T_{(x_1, y_1)}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & -x_1\cos\theta + y_1\sin\theta \\ \sin\theta & \cos\theta & -x_1\sin\theta - y_1\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_1(1-\cos\theta) + y_1\sin\theta \\ \sin\theta & \cos\theta & y_1(1-\cos\theta) - x_1\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

3

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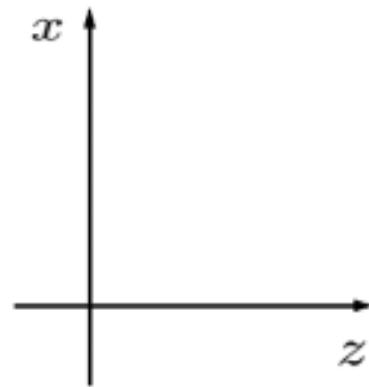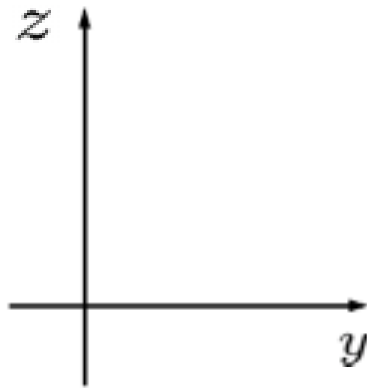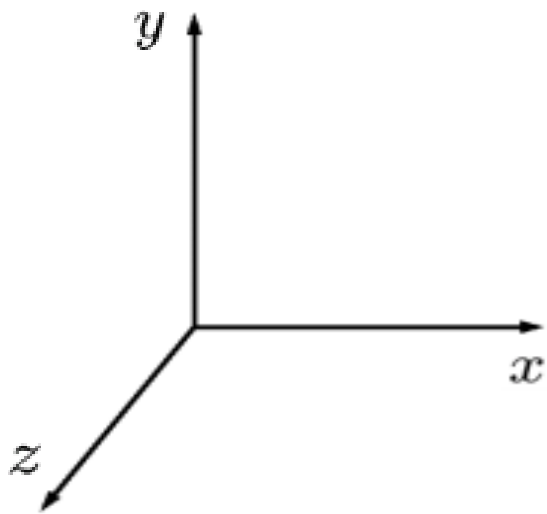
z-

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
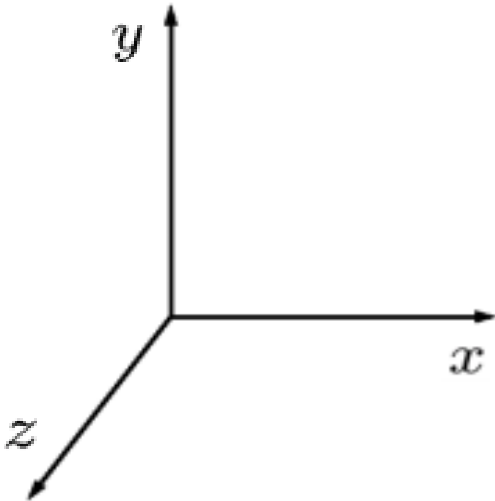
z-

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{bmatrix}$$

y-

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x

z

y
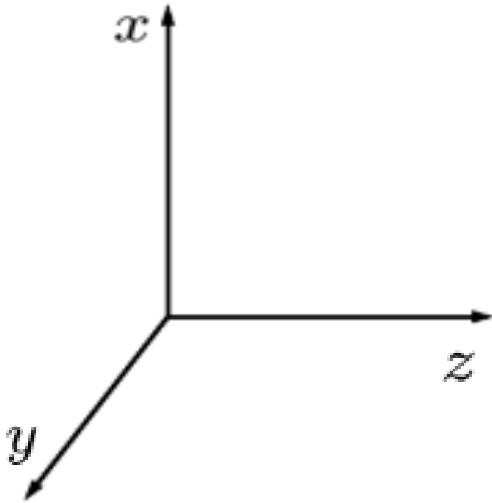
y-

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

X-

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

X-
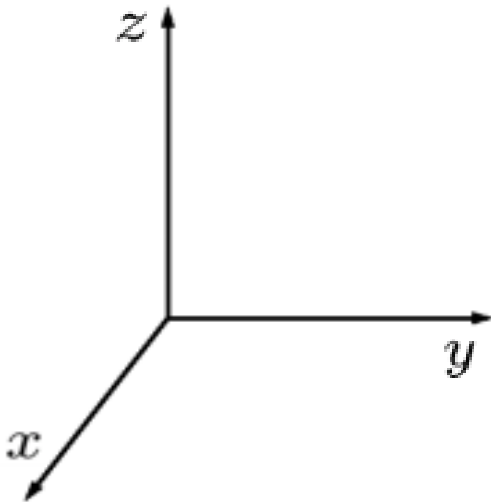
$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

3

$$
\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha x \\ \beta y \\ \gamma z \\ 1 \end{bmatrix}
$$

3

$$
\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix}
$$

3

$$
\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}
$$

3

$$\begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3

$$
\begin{bmatrix} b_x \\ b_y \\ b_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} c_x \\ c_y \\ c_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
$$

$$|a_{11}| = a_{11}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$= a_{11}a_{22} - a_{21}a_{12}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$+ a_{31} \begin{vmatrix} a_{12} & a_{22} \\ a_{22} & a_{23} \end{vmatrix}$$

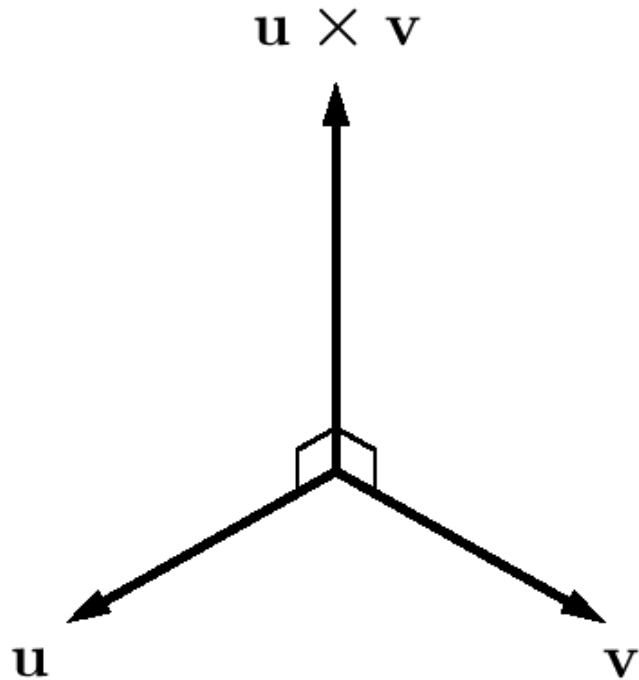$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 6 & 4 \\ 0 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} + 0 \begin{vmatrix} 2 & 6 \\ -1 & 0 \end{vmatrix}$$

$$= 1(12 - 0) - 3(4 + 4)$$

$$= -12$$

$$\begin{vmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix}$$

$$= (-3) \cdot 4 \cdot 5 = -60$$

$$\mathbf{u} \times \mathbf{v}$$

$$\begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathbf{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathbf{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}$$

$$+ \mathbf{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$\mathbf{e}_1 = (1, 0, 0)$$
$$\mathbf{e}_2 = (0, 1, 0)$$
$$\mathbf{e}_3 = (0, 0, 1)$$

$$\begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathbf{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathbf{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + \mathbf{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= (u_2 v_3 - u_3 v_2)\mathbf{e}_1 + (u_3 v_1 - u_1 v_3)\mathbf{e}_2$$

$$+ (u_1 v_2 - u_2 v_1)\mathbf{e}_3$$

$$= (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

3

$$(1, 1, 0) \times (3, 0, 0)$$

$$= \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ 1 & 1 & 0 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= \mathbf{e}_1 \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - \mathbf{e}_2 \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} + \mathbf{e}_3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= (-3) \cdot \mathbf{e}_3 = (0, 0, -3)$$

$$e_1 \times e_2 = e_3$$

$$e_2 \times e_3 = e_1$$

$$e_3 \times e_1 = e_2$$

$$e_2 \times e_1 = -e_3$$

$$e_3 \times e_2 = -e_1$$

$$e_1 \times e_3 = -e_2$$

$$(k\mathbf{u}) \times \mathbf{v} = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (k\mathbf{v}) = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$$

$$(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$$

$$\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \langle \mathbf{u}, \mathbf{w} \rangle \mathbf{v} - \langle \mathbf{u}, \mathbf{v} \rangle \mathbf{w}$$

$$\langle \mathbf{u}, \mathbf{v} \times \mathbf{w} \rangle = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$
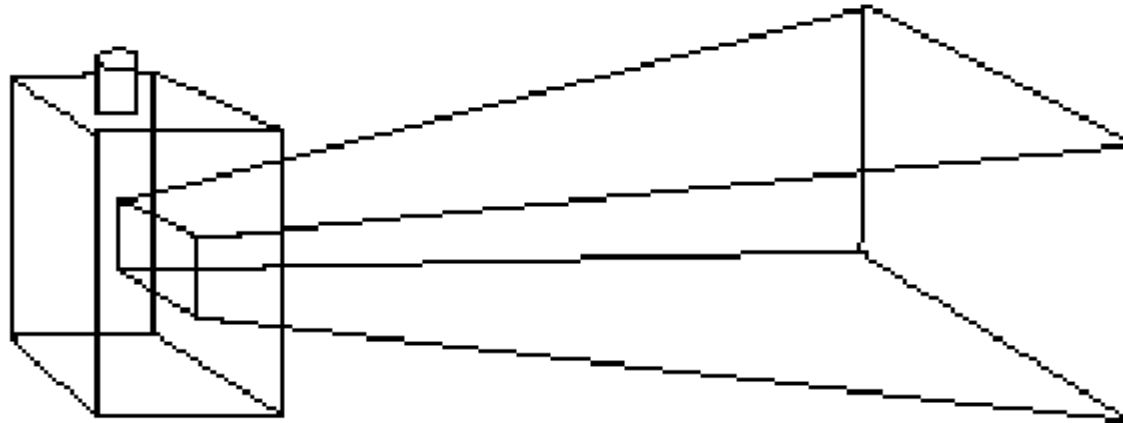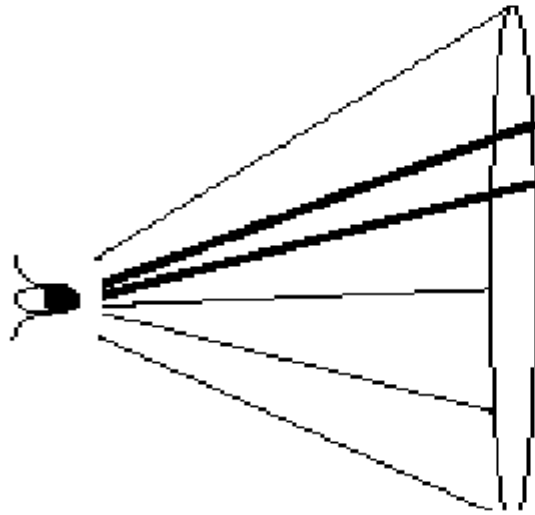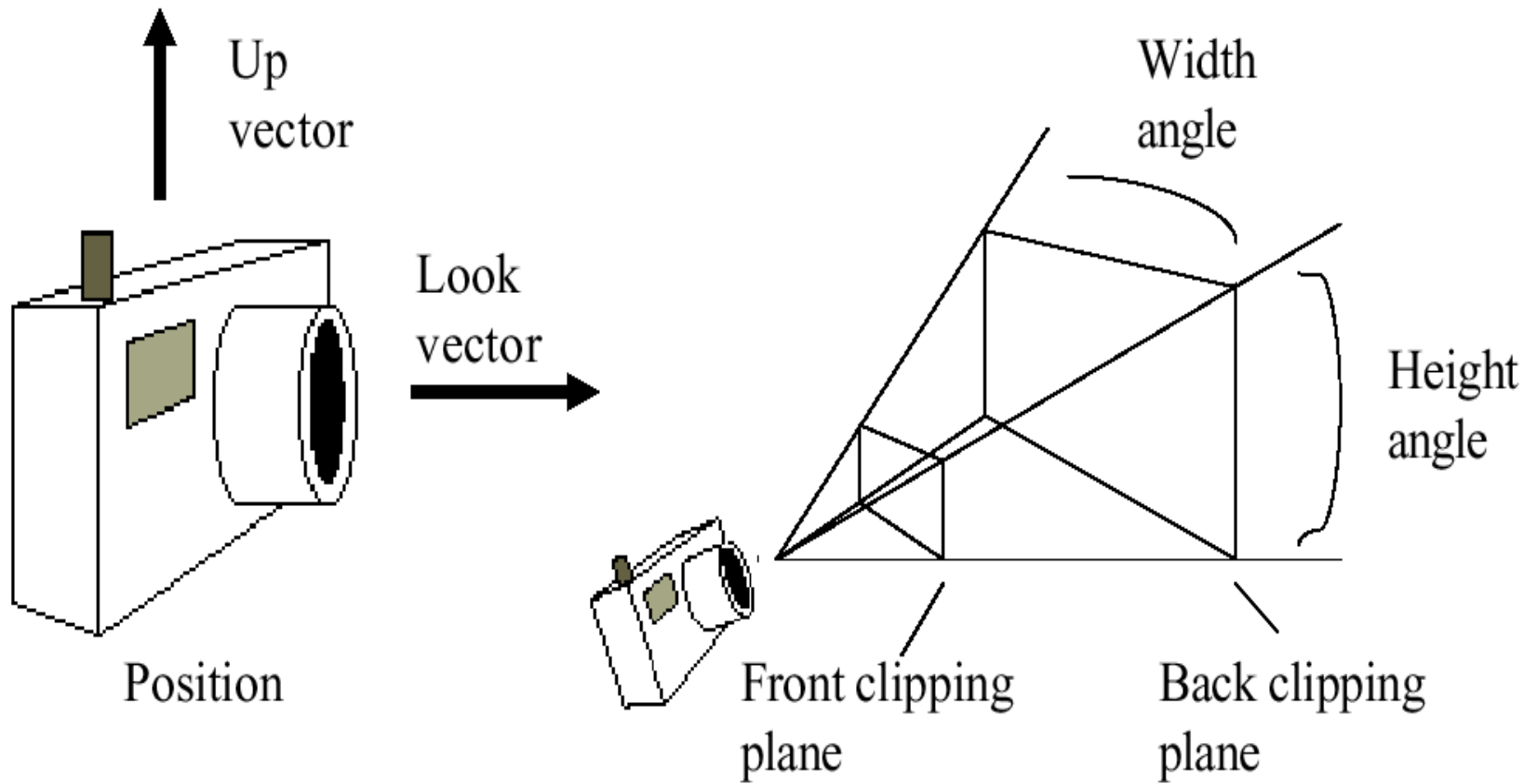
3장 Viewing

http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

3   View Volume

3   View Volume

Up vector

Look vector

Position

Width angle

Height angle

Front clipping plane

Back clipping plane

3   Clipping

Front clipping plane
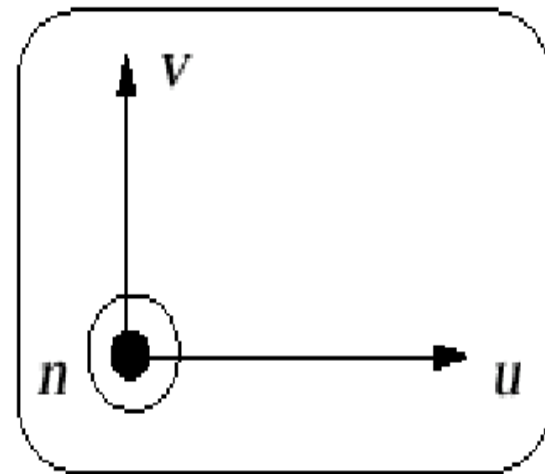
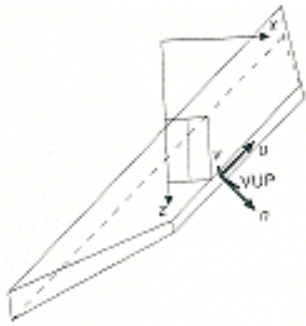Back clipping plane

Discarded   Rendered   Clipped   Discarded
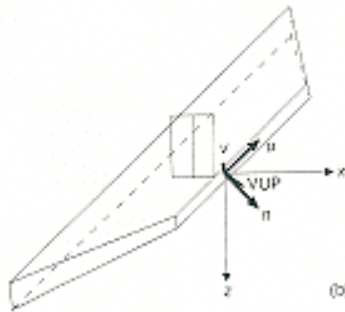
3   Viewing

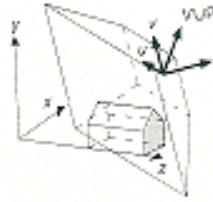

Arbitrary Perspective Frustum
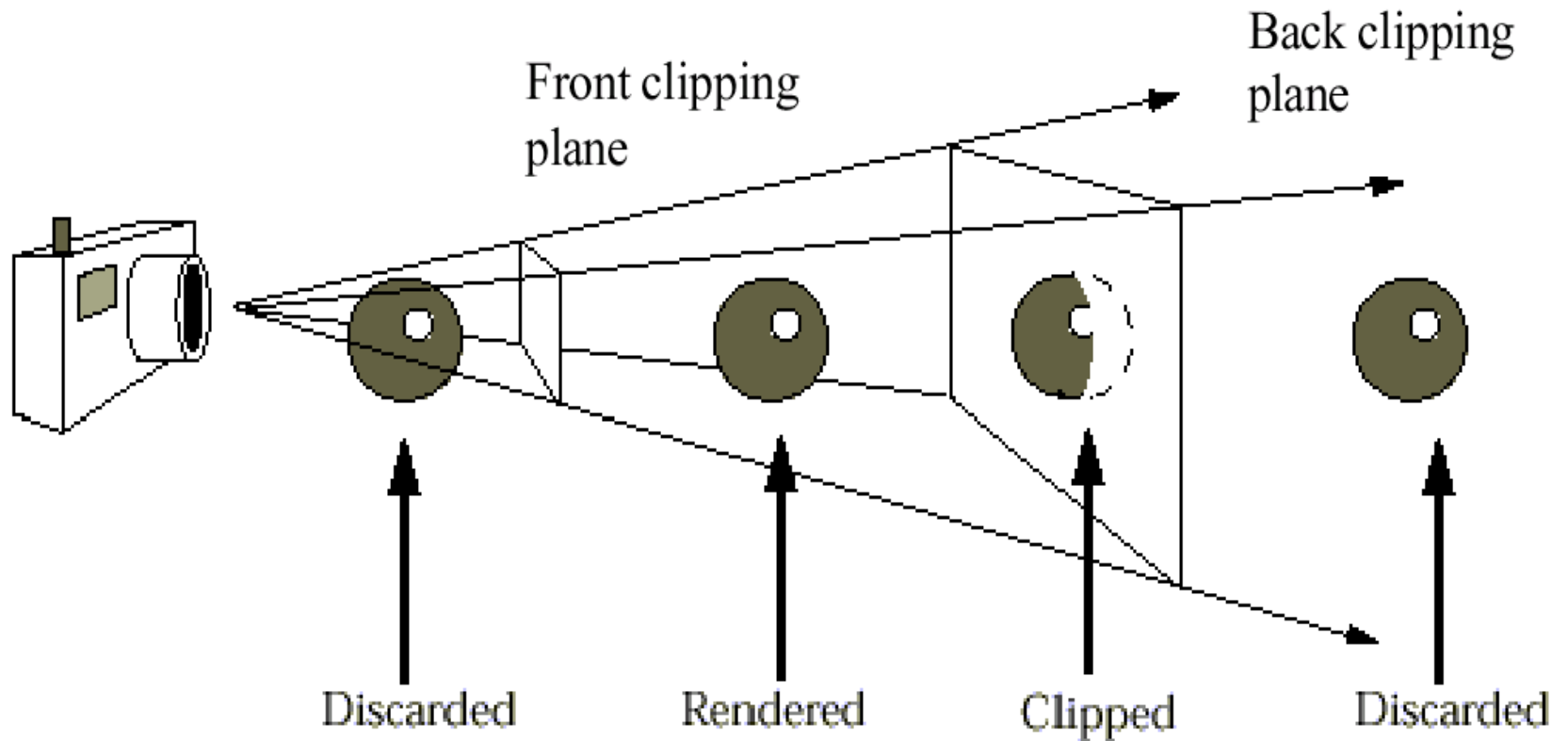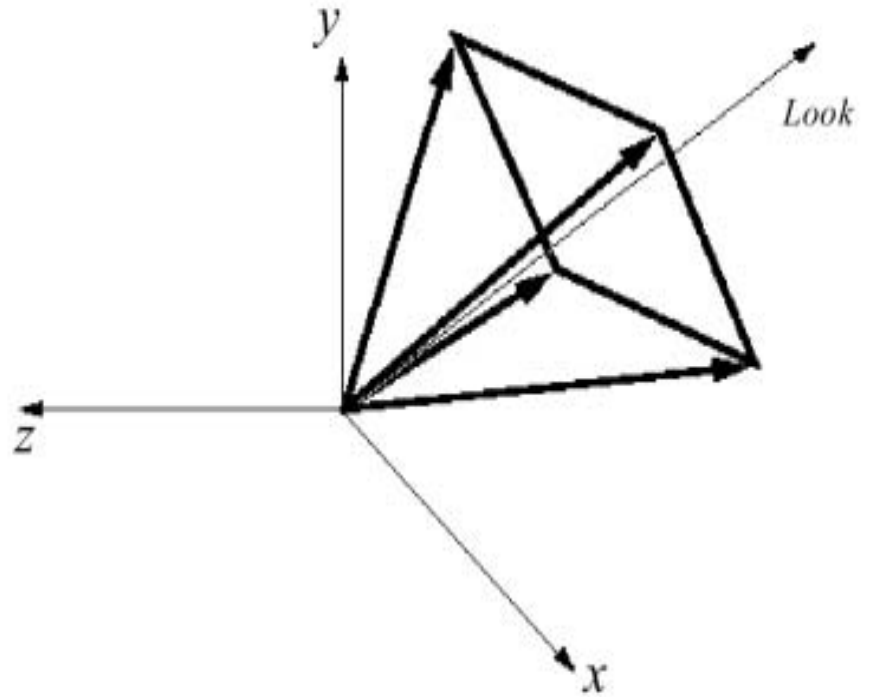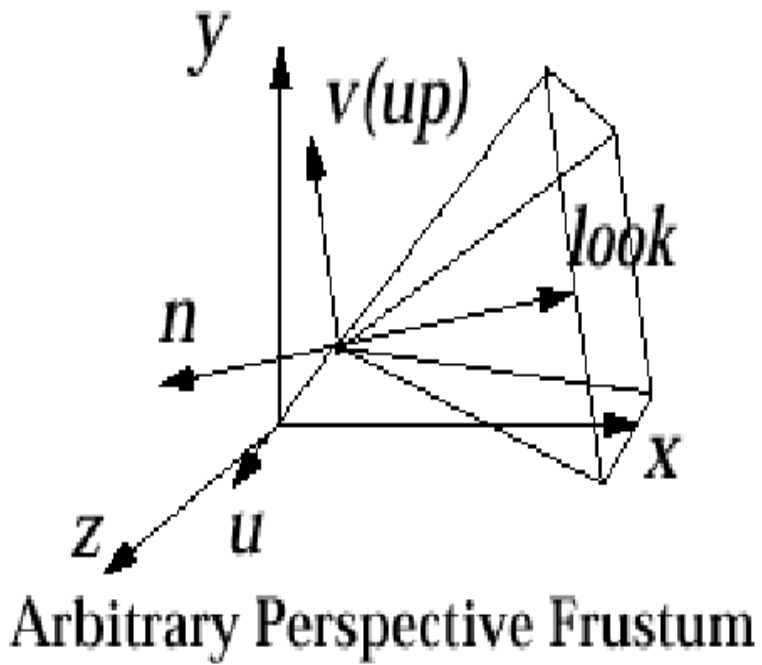
3 Viewing
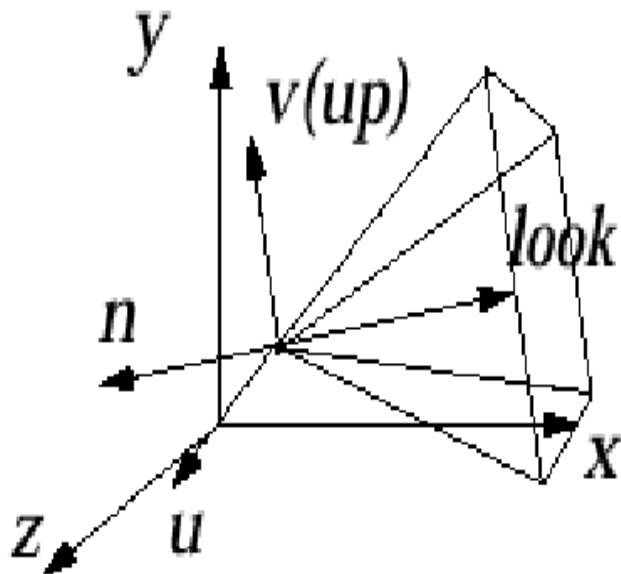
Arbitrary Perspective Frustum

Camera Coordinate System
(with *n* coming out of the paper)

(a)

(b)

(c)

(d)

(e)

(f)

$$\mathbf{x} - \mathbf{x_0} = (x - x_0, y - y_0)$$

$$\mathbf{n} = (a, b)$$

$$\mathbf{x} = (x, y)$$
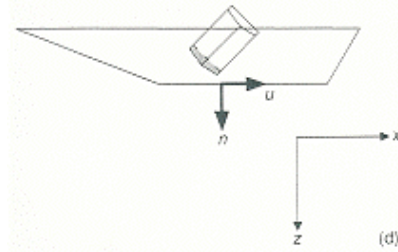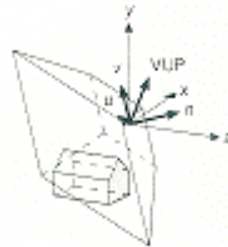
$$\mathbf{x_0} = (x_0, y_0)$$
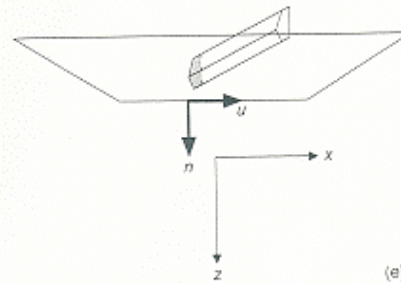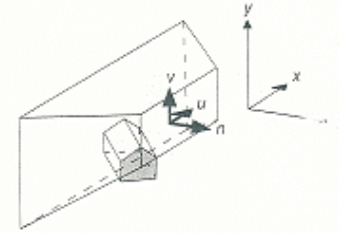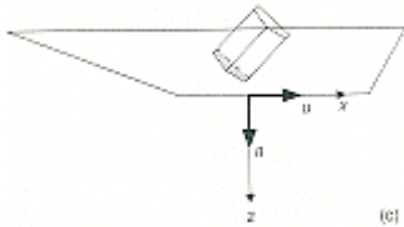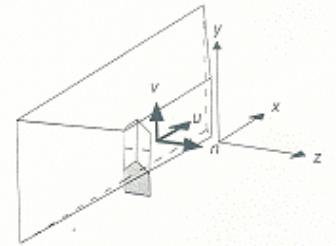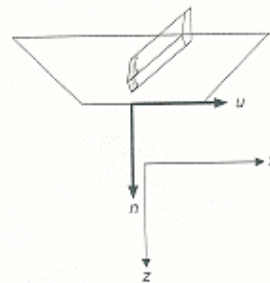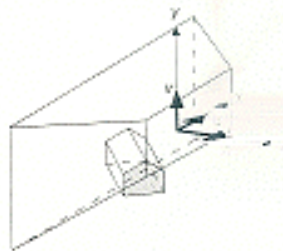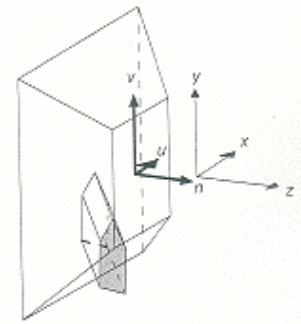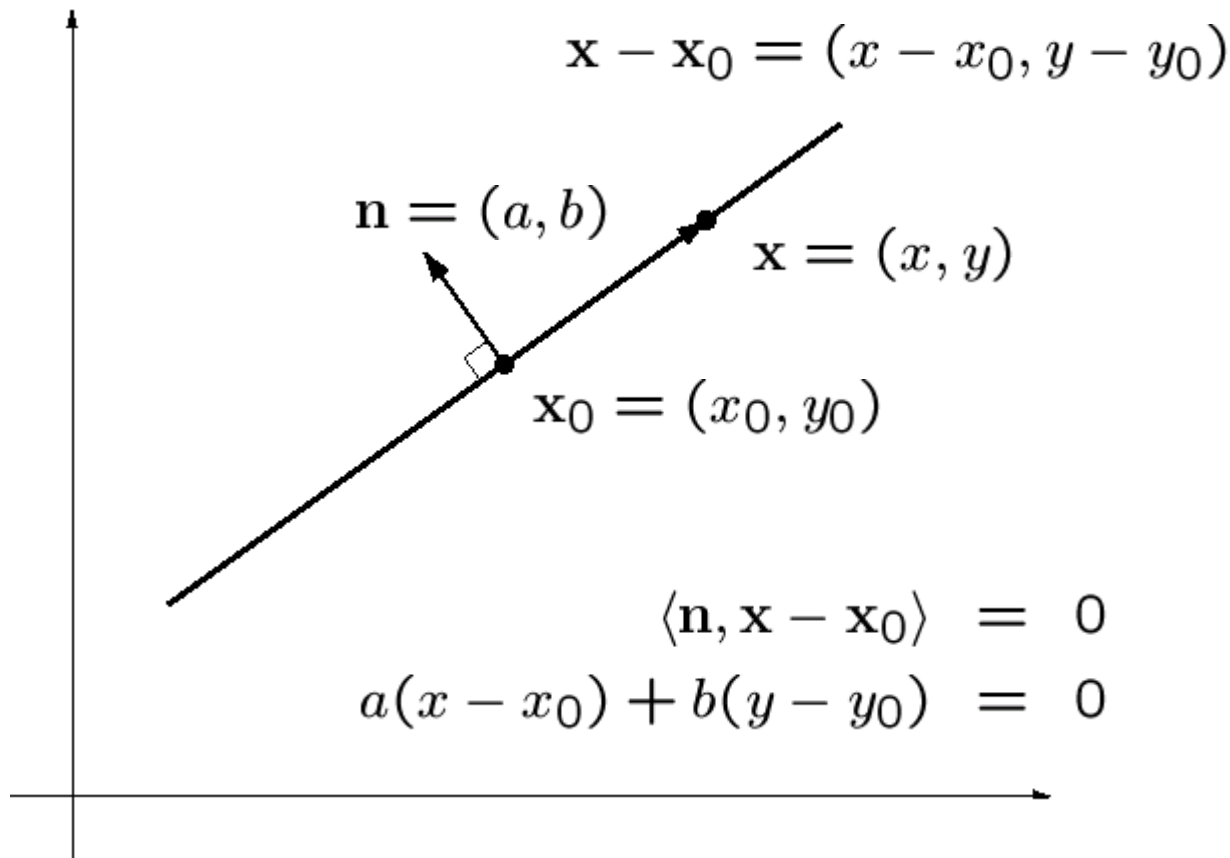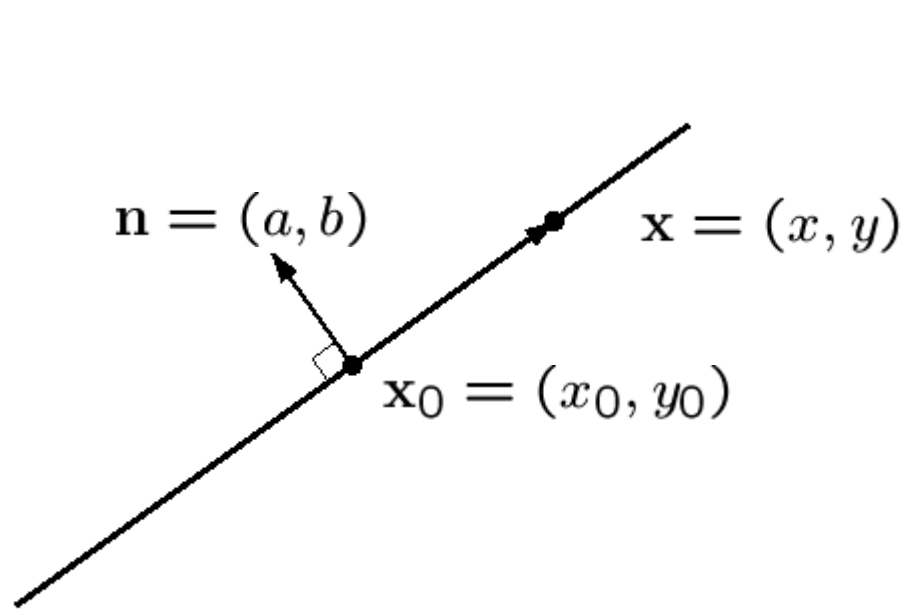
$$\langle \mathbf{n}, \mathbf{x} - \mathbf{x_0} \rangle = 0$$

$$a(x - x_0) + b(y - y_0) = 0$$

$$\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) = 0$$

$$ax + by - ax_0 - by_0 = 0$$

$$ax + by + c = 0$$

$$ax + by + c = 0$$

$$\mathbf{n} = (a, b)$$

$$\mathbf{x} = (x, y)$$

$$\mathbf{x}_0 = (x_0, y_0)$$

$$\mathbf{x}_1 = (x_1, y_1)$$

$$ax + by - ax_0 - by_0 = 0$$

$$ax + by + c = 0$$

$$ax_0 + by_0 + c \cdot 1 = 0$$

$$ax_1 + by_1 + c \cdot 1 = 0$$

$$c = \text{???}$$

$$\mathbf{n} = \text{???}$$
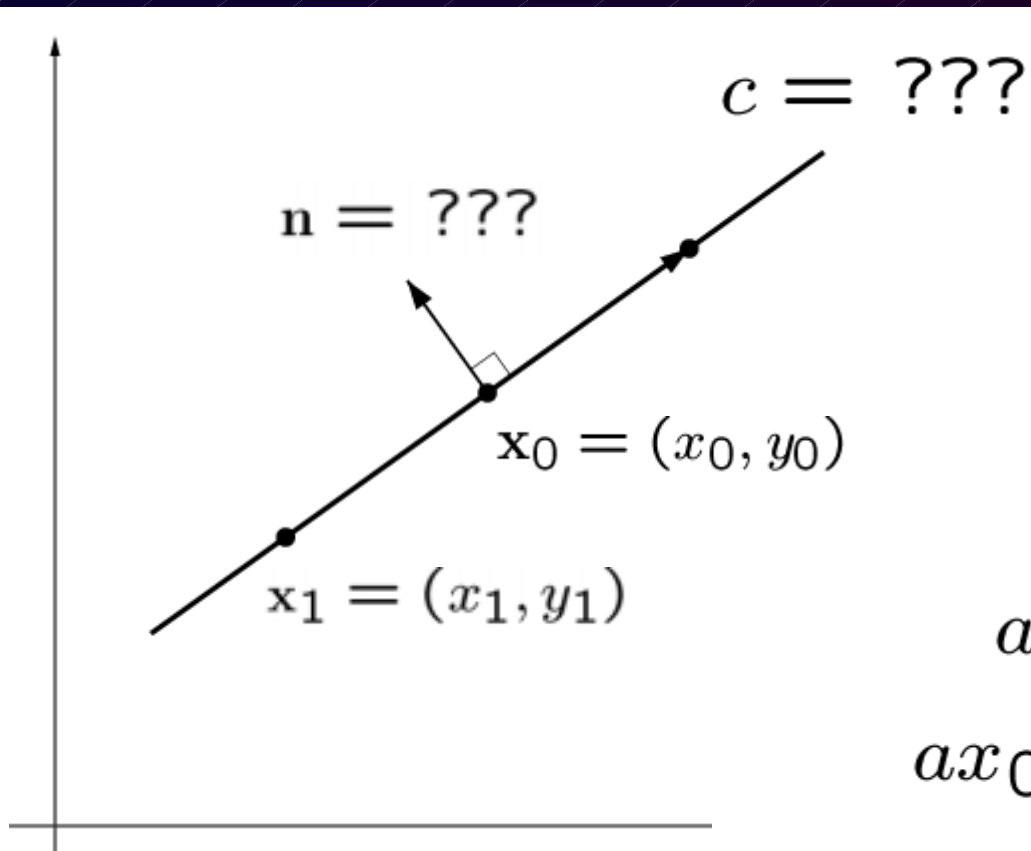
$$\mathbf{x}_0 = (x_0, y_0)$$

$$\mathbf{x}_1 = (x_1, y_1)$$

$$ax + by + c \cdot 1 = 0$$
$$ax_0 + by_0 + c \cdot 1 = 0$$
$$ax_1 + by_1 + c \cdot 1 = 0$$

$$\hat{\mathbf{n}} = \hat{\mathbf{x}}_0 \times \hat{\mathbf{x}}_1$$
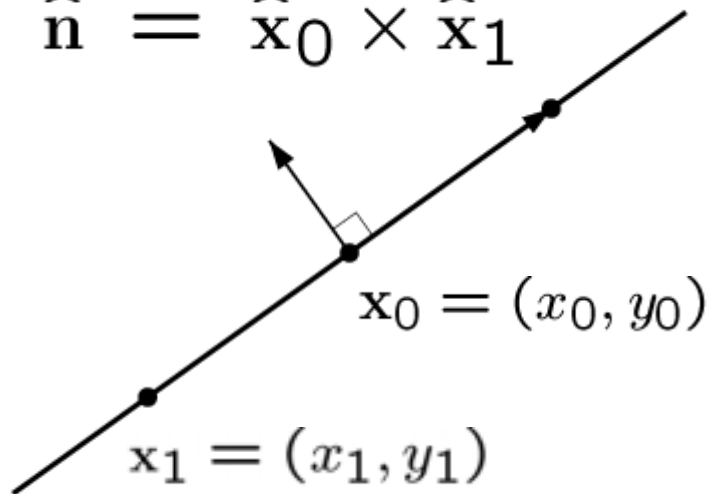
$$\mathbf{x}_0 = (x_0, y_0)$$

$$\mathbf{x}_1 = (x_1, y_1)$$

$$\hat{\mathbf{n}} = (a, b, c)$$

$$\hat{\mathbf{x}}_0 = (x_0, y_0, 1)$$

$$\hat{\mathbf{x}}_1 = (x_1, y_1, 1)$$

$$ax_0 + by_0 + c \cdot 1 = 0$$

$$ax_1 + by_1 + c \cdot 1 = 0$$

$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_0 \rangle = 0$$

$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_1 \rangle = 0$$

$$\hat{x}_0 = (2, 3, 1)$$

$$\hat{x}_1 = (4, 5, 1)$$

$$\hat{n} = \hat{x}_0 \times \hat{x}_1$$

$$= (2, 3, 1) \times (4, 5, 1)$$

$$= (-2, 2, -2)$$

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1$$

$$\hat{\mathbf{x}} = (x, y, 1)$$
$$\hat{\mathbf{n}}_0 = (a_0, b_0, c_0)$$
$$\hat{\mathbf{n}}_1 = (a_1, b_1, c_1)$$

$$\mathbf{x} = (x, y)$$
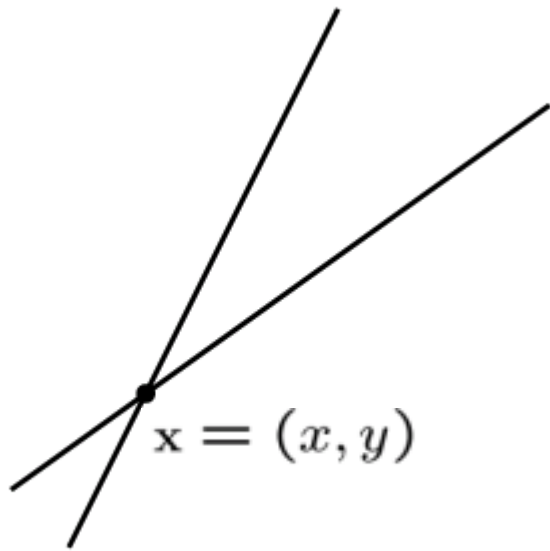
$$a_0 \cdot x + b_0 \cdot y + c_0 \cdot 1 = 0$$
$$a_1 \cdot x + b_1 \cdot y + c_1 \cdot 1 = 0$$
$$\langle \hat{\mathbf{n}}_0, \hat{\mathbf{x}} \rangle = 0$$
$$\langle \hat{\mathbf{n}}_1, \hat{\mathbf{x}} \rangle = 0$$

$$x + y - 1 = 0$$

$$x - 2y + 1 = 0$$

$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{n}}_0 = (1, 1, -1)$$

$$\hat{\mathbf{n}}_1 = (1, -2, 1)$$

$$\mathbf{x} = (x, y)$$
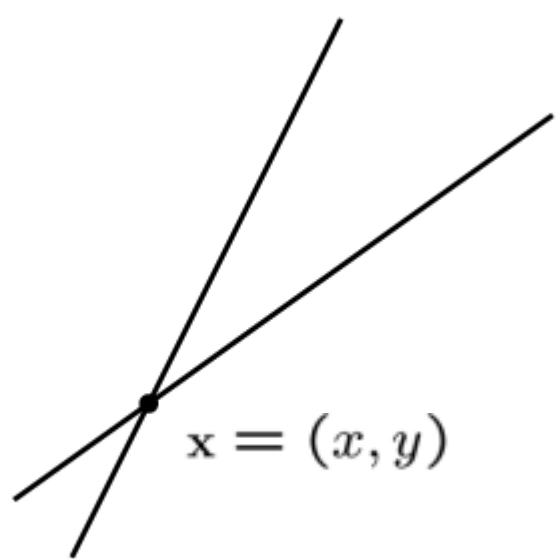
$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1$$

$$= (1, 1, -1) \times (1, -2, 1)$$

$$= (-1, -2, -3)$$

$$= \left(\frac{1}{3}, \frac{2}{3}, 1 \right)$$

$$\mathbf{n} = (a, b, c)$$

$$\mathbf{x} - \mathbf{x}_0 = (x - x_0, y - y_0, z - z_0)$$

$$0 = \langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle$$

$$0 = a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$0 = ax + by + cz - ax_0 - by_0 - cz_0$$

$$0 = ax + by + cz + d$$

$$\widehat{\mathbf{n}} = (a, b, c, d)$$

$$\widehat{\mathbf{x}}_0 = (x_0, y_0, z_0, 1)$$

$$\widehat{\mathbf{x}}_1 = (x_1, y_1, z_1, 1)$$

$$\widehat{\mathbf{x}}_2 = (x_2, y_2, z_2, 1)$$

$$ax_0 + by_0 + cz_0 + d \cdot 1 = 0$$

$$ax_1 + by_1 + cz_1 + d \cdot 1 = 0$$

$$ax_2 + by_2 + cz_2 + d \cdot 1 = 0$$

$$ax_0 + by_0 + cz_0 + d \cdot 1 = 0$$

$$ax_1 + by_1 + cz_1 + d \cdot 1 = 0$$

$$ax_2 + by_2 + cz_2 + d \cdot 1 = 0$$

$$\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_0 \rangle = 0$$

$$\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_1 \rangle = 0$$

$$\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_2 \rangle = 0$$
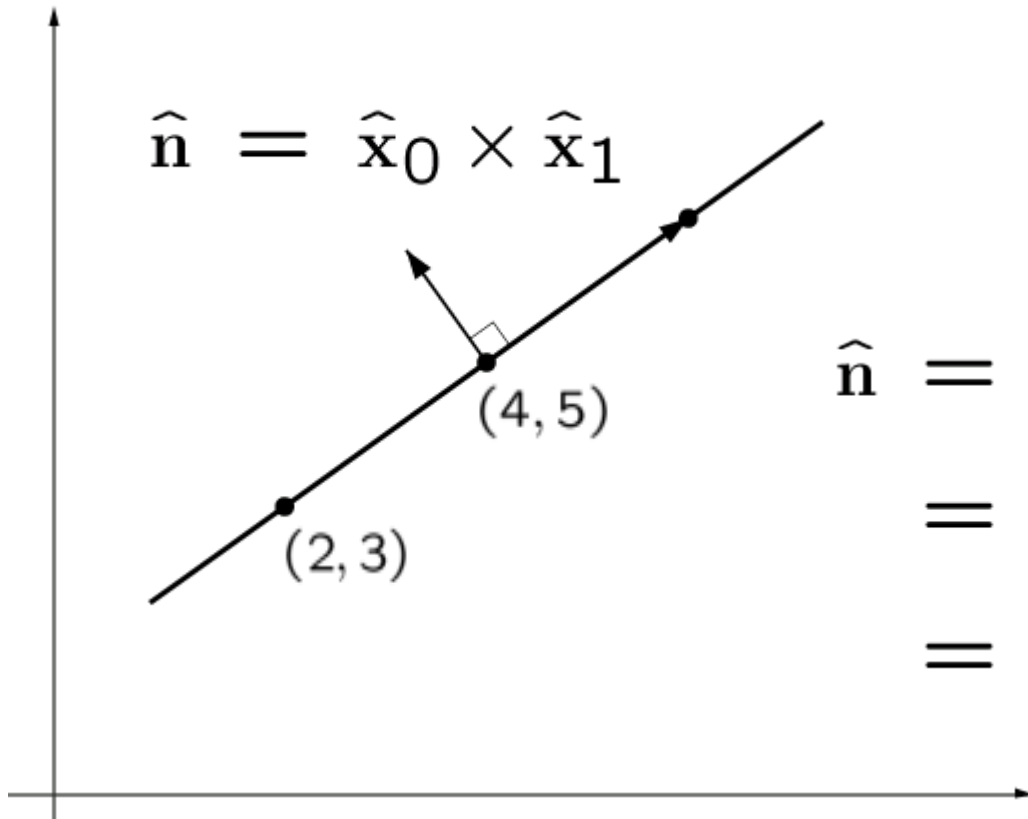
$$\widehat{\mathbf{n}} = \widehat{\mathbf{x}}_0 \wedge \widehat{\mathbf{x}}_1 \wedge \widehat{\mathbf{x}}_2$$

Wedge Product

$$\hat{\mathbf{n}} = \hat{\mathbf{x}}_0 \wedge \hat{\mathbf{x}}_1 \wedge \hat{\mathbf{x}}_2$$

$$= \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \end{bmatrix}$$

$$\hat{\mathbf{x}} = (x, y, z, w)$$

$$\hat{\mathbf{n}}_0 = (a_0, b_0, c_0, d_0)$$

$$\hat{\mathbf{n}}_1 = (a_1, b_1, c_1, d_1)$$

$$\hat{\mathbf{n}}_2 = (a_2, b_2, c_2, d_2)$$

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \wedge \hat{\mathbf{n}}_1 \wedge \hat{\mathbf{n}}_2$$

$$\hat{\mathbf{n}} = (a, b, c)$$

$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{v}} = (v_x, v_y, 1)$$

$$\hat{\mathbf{x}}' = (x', y', 1)$$

$$= \hat{\mathbf{n}} \times (\hat{\mathbf{x}} \times \hat{\mathbf{v}})$$

$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \, \hat{\mathbf{v}}$$

$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \hat{\mathbf{v}} \, \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y + c & -bv_x & -cv_x \\ -av_y & av_x + c & -cv_y \\ -a & -b & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
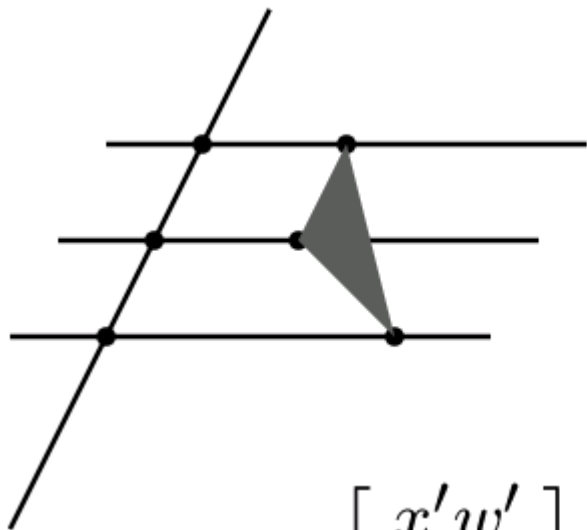
$$\hat{\mathbf{x}}' \;=\; \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \hat{\mathbf{v}} \, \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\longleftarrow \quad \hat{\mathbf{v}} = (v_x, v_y, 0)$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y & -bv_x & -cv_x \\ -av_y & av_x & -cv_y \\ 0 & 0 & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

Perspective and parallel three-dimensional projections

3

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \hat{\mathbf{v}} \, \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$
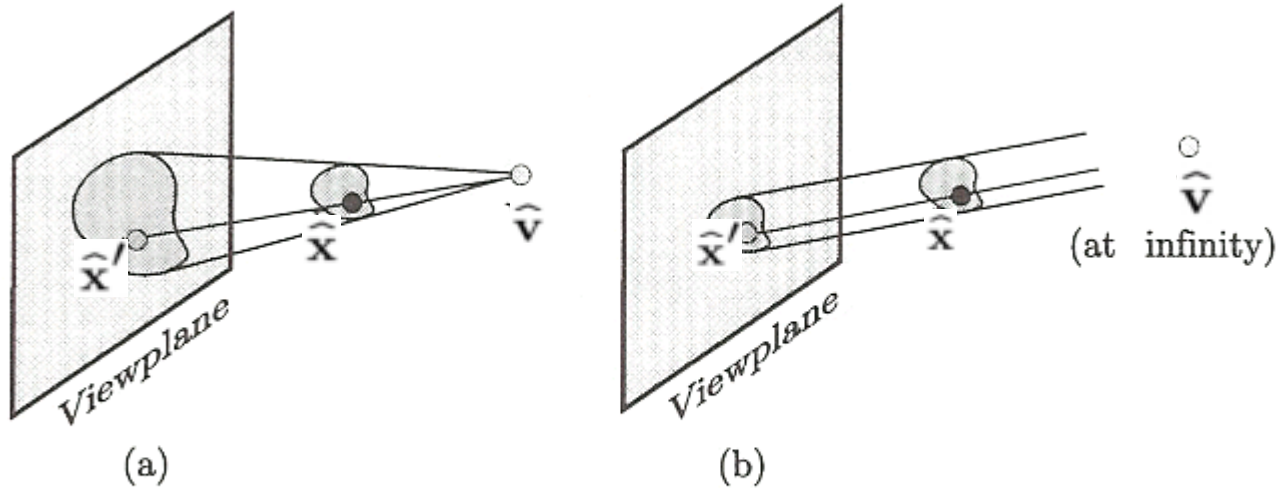
(Case I): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle = 0$,

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} = \hat{\mathbf{x}}$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\hat{\mathbf{x}}' = \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}}$$

$$0 = \langle \hat{\mathbf{n}}, \hat{\mathbf{x}}' \rangle$$

$$0 = \langle \hat{\mathbf{n}}, \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \rangle$$

$$\hat{x}' = \langle \hat{n}, \hat{v} \rangle \hat{x} - \hat{v} \langle \hat{n}, \hat{x} \rangle$$

(Case II): $\langle \hat{n}, \hat{x} \rangle \neq 0$,

$$\hat{x}' = \alpha \hat{x} + \beta \hat{v}$$

$$0 = \langle \hat{n}, \hat{x}' \rangle$$

$$0 = \langle \hat{n}, \alpha \hat{x} + \beta \hat{v} \rangle$$

$$0 = \alpha \langle \hat{n}, \hat{x} \rangle + \beta \langle \hat{n}, \hat{v} \rangle$$

$$\alpha = -\beta \frac{\langle \hat{n}, \hat{v} \rangle}{\langle \hat{n}, \hat{x} \rangle}$$

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\alpha = -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle}$$

$$\hat{\mathbf{x}}' = -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle} \hat{\mathbf{x}} + \beta \hat{\mathbf{v}}$$

$$= \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle} \hat{\mathbf{x}} - \hat{\mathbf{v}}$$

$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \hat{\mathbf{v}}$$

$$\widehat{\mathbf{x}}' \;=\; \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$
\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} =
\begin{bmatrix} a & b & c & d \end{bmatrix}
\begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

$$
- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix}
\begin{bmatrix} a & b & c & d \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
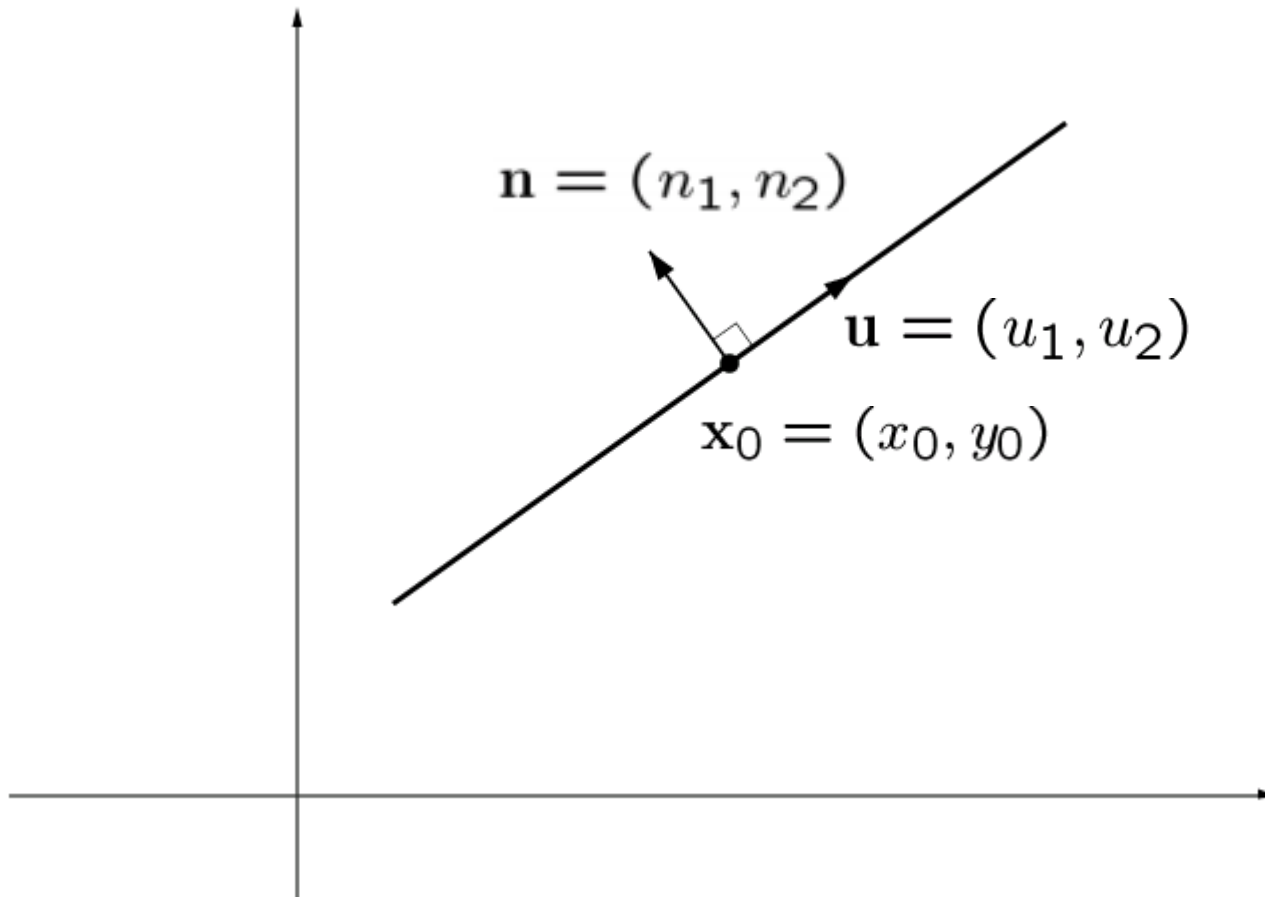
$$\|\mathbf{u}\| = \|\mathbf{n}\| = 1$$

$$\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{n} = (n_1, n_2)$$

$$\mathbf{u} = (u_1, u_2)$$

$$\mathbf{x}_0 = (x_0, y_0)$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = (x_0, y_0, z_0)$$
$$\mathbf{u} = (u_1, u_2, u_3)$$
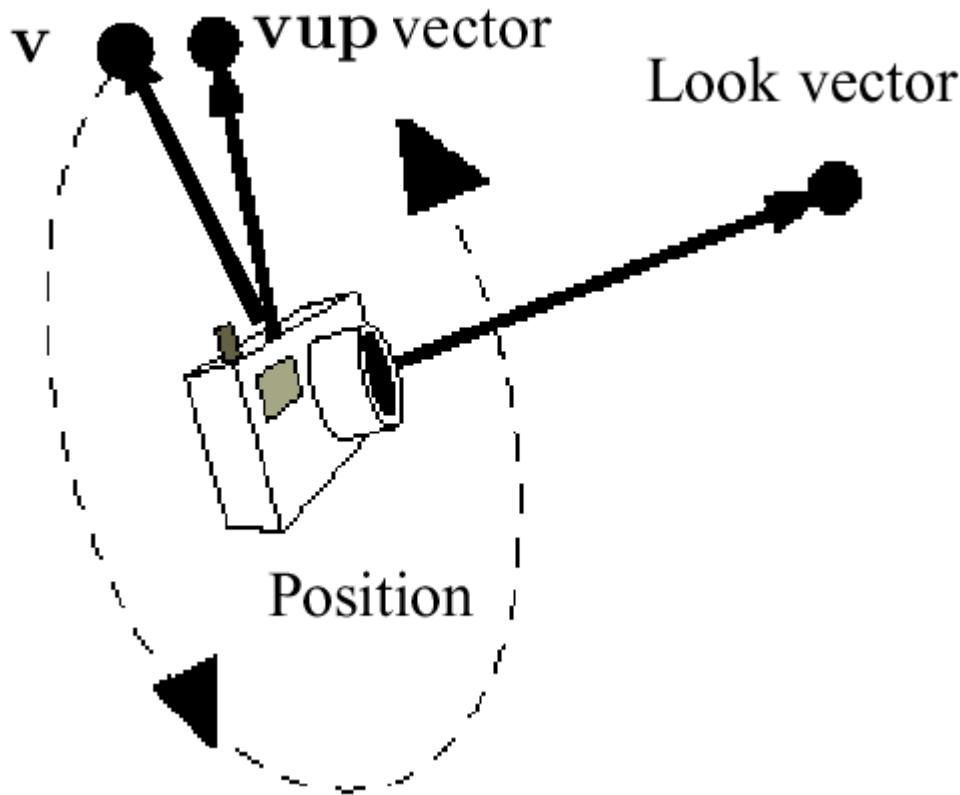$$\mathbf{v} = (v_1, v_2, v_3)$$
$$\mathbf{n} = (n_1, n_2, n_3)$$

$$\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{n} \rangle = \langle \mathbf{v}, \mathbf{n} \rangle = 0$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{n}\| = 1$$

$$
\begin{bmatrix}
u_1 & u_2 & u_3 & 0 \\
v_1 & v_2 & v_3 & 0 \\
n_1 & n_2 & n_3 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & -x_0 \\
0 & 1 & 0 & -y_0 \\
0 & 0 & 1 & -z_0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$u = \frac{vup \times n}{\|vup \times n\|}$$

$$v = n \times u$$

3   Viewing

- 3   2   viewing

- Viewing   (x0,y0,z0)

- U, V, N   X, Y, Z

- ·   XY
              .   ▯