Quiz #4 (CSE 400.001)

Wednesday, November 3, 2004

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1. (10 points) Solve $A\mathbf{x} = \mathbf{b}_1$ and $A\mathbf{x} = \mathbf{b}_2$, compare the solutions, and comment. Compute the condition number of $A$.

$$A = \begin{bmatrix} 3.0 & 1.7 \\ 1.7 & 1.0 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 4.7 \\ 2.7 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 4.7 \\ 2.71 \end{bmatrix}$$

$$\mathbf{x}_1 = A^{-1}\mathbf{b}_1 = \frac{1}{0.11}\begin{bmatrix} 1.0 & -1.7 \\ -1.7 & 3.0 \end{bmatrix}\begin{bmatrix} 4.7 \\ 2.7 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad +2$$

$$\mathbf{x}_2 = A^{-1}\mathbf{b}_2 = \frac{1}{0.11}\begin{bmatrix} 1.0 & -1.7 \\ -1.7 & 3.0 \end{bmatrix}\begin{bmatrix} 4.7 \\ 2.71 \end{bmatrix} = \begin{bmatrix} 0.845 \\ 1.273 \end{bmatrix} \quad +2$$

$$\mathbf{x}_1 - \mathbf{x}_2 = \begin{bmatrix} 0.154 \\ -0.273 \end{bmatrix}, \quad \mathbf{b}_1 - \mathbf{b}_2 = \begin{bmatrix} 0.0 \\ -0.01 \end{bmatrix}$$

+2

$A^{-1}$ has elements in the range of $-17 \leq a_{ij} \leq 30$,

The difference $\mathbf{x}_1 - \mathbf{x}_2$ in the solution $\mathbf{x}$ is magnified by 17 ~ 30 times from the difference $\mathbf{b}_1 - \mathbf{b}_2$. +2

$$\kappa(A) = \|A\|_1 \cdot \|A^{-1}\|_1 \approx 4.7 * 42.73 \approx 200.83$$

$$= \|A\|_\infty \cdot \|A^{-1}\|_\infty \approx 4.7 * 42.73 \approx 200.83$$

+2

2. (15 points) Given an $m \times n$ matrix $A = [a_{ij}]$, let

$$\mathbf{x}_i = (sgn(a_{i1}), \cdots, sgn(a_{in}))^T,$$

where $sgn(a) = 1$ if $a \geq 0$ and $sgn(a) = -1$ if $a < 0$.

(a) (10 points) Show that

$$\|A\mathbf{x}_i\|_\infty \geq \sum_{j=1}^{n} |a_{ij}|$$

(b) (5 points) Using the above result, show that

$$\max_{\|\mathbf{x}\|_\infty = 1} \|A\mathbf{x}\|_\infty \geq \max_{i=1,\cdots,m} \left( \sum_{j=1}^{n} |a_{ij}| \right)$$

(a)

$$A\mathbf{x}_i = \begin{bmatrix} \vdots \\ |a_{i1}| + \cdots + |a_{in}| \\ \vdots \end{bmatrix} \leftarrow i\text{-th element}$$

+7

$$\|A\mathbf{x}_i\|_\infty \geq |\,i\text{-th element}\,| = \sum_{j=1}^{n} |a_{ij}|$$

+3

(b) $$\|\mathbf{x}_i\|_\infty = \max_{j=1,\cdots,n} |sgn(a_{ij})| = \max|\pm 1| = 1$$

+2

$$\max_{\|\mathbf{x}\|_\infty = 1} \|A\mathbf{x}\|_\infty \geq \max_{i=1,\cdots,m} \|A\mathbf{x}_i\|_\infty$$

+2

$$\geq \max_{i=1,\cdots,m} \left( \sum_{j=1}^{n} |a_{ij}| \right)$$

+1