

# Quiz #6 (CSE 400.001)

Wednesday, November 10, 2004

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1. (10 points) Using  $h = 1/2$  and  $k = 2/3$ , approximate the solution to the following elliptic equation

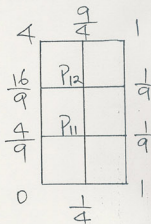
$$u_{xx} + 4u_{yy} = 9, \quad 0 < x < 1, \quad 0 < y < 2$$

with boundary conditions:

$$\begin{aligned} u(x, 0) &= x^2, & u(x, 2) &= (x-2)^2, & 0 \leq x \leq 1; \\ u(0, y) &= y^2, & u(1, y) &= (y-1)^2, & 0 \leq y \leq 2. \end{aligned}$$

Set up a system of linear equations.

$i$	$j$	$x_i$	$y_j$	$u(x_i, y_j)$
1	1	1/2	2/3	
1	2	1/2	4/3	



$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + 4 \cdot \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 9 \quad (+3)$$

$$4(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) + 9(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) = 9 \quad (+2)$$

$$4u_{i+1,j} - 26u_{i,j} + 4u_{i-1,j} + 9u_{i,j+1} + 9u_{i,j-1} = 9 \quad (+1)$$

$$\begin{cases} P_{11}: -26u_{11} + 9u_{12} = \frac{163}{36} & (+2) \\ P_{12}: -26u_{12} + 9u_{11} = -\frac{671}{36} & (+2) \end{cases}$$

2. (15 points) Consider the following hyperbolic equation

$$u_{tt} = u_{xx} + 100, \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 0.4,$$

with initial and boundary conditions

$$u(x, 0) = x^3, \quad u_t(x, 0) = x^2; \quad u_x(0, t) = t^2, \quad u(1, t) = (1+t)^3,$$

Approximate the solution to above equation with  $h = k = 0.2$ , for  $0 \leq t \leq 0.4$ .

(a) (5 points) Represent  $u_{i,j+1}$  in terms of  $u_{i-1,j}, u_{i,j}, u_{i+1,j}, u_{i,j-1}$ .

(b) (5 points) Represent  $u_{i,1}$  in terms of  $u_{i-1,0}, u_{i,0}, u_{i+1,0}$ .

(c) (5 points) Represent  $u_{0,j+1}$  in terms of  $u_{0,j}, u_{1,j}, u_{0,j-1}$ .

$$(a) \frac{1}{k^2} [u_{i,j+1} - 2u_{i,j} + u_{i,j-1}] = \frac{1}{h^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}] + 100$$

$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j} + 4 \quad (+2) \quad (+3)$$

$$(b) u_{i,1} - u_{i,-1} = 2k (h^2 i^2) = 0.016 i^2 \quad (+2)$$

$$u_{i,1} = u_{i+1,0} + u_{i-1,0} - u_{i,0} + 0.016 i^2 + 4 \quad (+2)$$

$$u_{i,1} = \frac{1}{2} (u_{i+1,0} + u_{i-1,0}) + 0.008 i^2 + 2 \quad (+1)$$

$$(c) u_{1,j} - u_{-1,j} = 2k (k^2 j^2) = 0.016 j^2 \quad (+2)$$

$$u_{0,j+1} = u_{1,j} + u_{-1,j} - u_{0,j} + 4$$

$$= u_{1,j} + u_{-1,j} - 0.016 j^2 - u_{0,j} + 4 \quad (+3)$$

$$= 2u_{1,j} - u_{0,j} - 0.016 j^2 + 4$$