Quiz #3 (CSE 400.001)

Tuesday, April 22, 2003

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1. (7 points) Use $x_0 = 1.2$ and $x_1 = 1.153$ in solving the following equation by Newton's method

$$x^5 - 2 = 0.$$

How many additional iterations are necessary to produce the solution to 10D accuracy?

Solution:

$$\frac{f''(s)}{2f'(s)} \approx \frac{f''(x_1)}{2f'(x_1)} = \frac{20x_1^3}{10x_1^4} = \frac{2}{x_1} \approx 1.735$$

$$|\epsilon_{n+1}| \approx 1.735\epsilon_n^2 \approx 1.735^3\epsilon_{n-1}^4 \approx 1.735^{2^{n+1}-1}\epsilon_0^{2^{n+1}} \leq 5 \cdot 10^{-11}$$

$$\epsilon_1 - \epsilon_0 = (\epsilon_1 - s) - (\epsilon_0 - s) = -x_1 + x_0 \approx 0.047$$

$$\epsilon_1 \approx \epsilon_0 + 0.047 \approx -1.735\epsilon_0^2$$

$$1.735\epsilon_0^2 + \epsilon_0 + 0.047 \approx 0$$

$$\epsilon_0 \approx -0.05163$$

$$\begin{aligned}
n = 1: &\quad 1.735^3 \cdot 0.05163^4 \approx 3.711 \cdot 10^{-5} > 5 \cdot 10^{-11} \\
n = 2: &\quad 1.735^7 \cdot 0.05163^8 \approx 2.390 \cdot 10^{-9} > 5 \cdot 10^{-11} \\
n = 3: &\quad 1.735^{15} \cdot 0.05163^{16} < 10^{-16} < 5 \cdot 10^{-11}
\end{aligned}$$

Hence, $n = 3$ additional iterations are necessary.

2. (4 points) Interpolate

$$f_0 = f(0) = 0,\ f_1 = f(1) = 9,\ f_2 = f(2) = 9,\ f_3 = f(3) = 0$$

by the cubic spline satisfying $k_0 = 9$ and $k_3 = -9$.

Solution:

$$\begin{cases} k_0 + 4k_1 + k_2 = 3 \cdot (9) = 27 \\ k_1 + 4k_2 + k_3 = 3 \cdot (-9) = -27 \end{cases} \Longrightarrow \begin{cases} 4k_1 + k_2 = 18 \\ k_1 + 4k_2 = -18 \end{cases} \Longrightarrow k_1 = 6, \quad k_2 = -6$$

$$\begin{cases} p_0(x) &=& -3x^3 + 3x^2 + 9x, \quad \text{for } 0 \le x \le 1 \\ p_1(x) &=& -6(x-1)^2 + 6(x-1) + 9, \quad \text{for } 1 \le x \le 2 \\ p_2(x) &=& 3(x-2)^3 - 6(x-2)^2 - 6(x-2) + 9, \quad \text{for } 2 \le x \le 3 \end{cases}$$

3. (4 points) Compute the following integral using the Gauss quadrature with $n = 5$.

$$\int_1^5 \frac{2x}{1+x^2} dx$$

Solution:

$$x = 2t + 3 \quad \Rightarrow \quad dx = 2dt$$

$$\begin{aligned} & \int_{-1}^{1} \frac{4t+6}{4t^2+12t+10} \cdot 2dt \\ = & \int_{-1}^{1} \frac{4t+6}{2t^2+6t+5} dt \\ = & \cdots \end{aligned}$$