Quiz #2 (CSE4190.410)

October 6, 2004 (Wednesday)

1. (2 points) What are the most serious shortcomings of vector display?

   (1) It cannot display areas filled with solid colors or patterns.

   (2) It is very expensive. Its manufacturing cost is considerably higher than raster display.

2. (2 points) What had been the main obstacles of raster display in the past?

   (1) Large memory requirement for frame buffer.

   (2) Much computation time for rasterization.

3. (2 points) In which applications/situations is the color lookup table useful?

   (1) Simple animations such as bouncing balls and liquid flows, etc.

   (2) Color display systems with limited memory space.

4. (4 points) Three lines $L_i : a_i x + b_i y + c_i = 0$, $i = 1, 2, 3$, share a common intersection point if and only if

   $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

   Explain why.

   The intersection point of two lines L_2 and L_3 is given as $(a_2, b_2, c_2) \times (a_3, b_3, c_3)$ in homogeneous coordinates. This point is located on the line L_1 if and only if

   $$\langle (a_1, b_1, c_1), (a_2, b_2, c_2) \times (a_3, b_3, c_3) \rangle = 0,$$

   which is equivalent to the above equation.

5. (4 points) In a scan-line polygon-fill algorithm, the change in y coordinates between two scan lines is $y_{k+1} = y_k + 1$, and the corresponding x intercepts along a boundary edge change based on

   $$x_{k+1} = x_k + \frac{\Delta x}{\Delta y}.$$

   We can use integer arithmetics so as to round the x intercepts to the nearest pixel value. We initialize a counter to 0 and increment the counter by $2\Delta x$ at each step. Then how should we proceed?

   Compare the counter with Δy. If it is greater than or equal to Δy, we increase the x value by 1 and decrement the counter by $2\Delta y$, and continue the same procedure.

6. (6 points) Many rasterization algorithms can be extended to similar voxelization algorithms for 3D volume graphics.

   (a) (2 points) Discuss how to extend Bresenham's line-drawing algorithm to a voxelization algorithm for generating a 3D line segment connecting two points (x_0, y_0, z_0) and (x_1, y_1, z_1), where the coordinate values are given as integers.

   We may assume $|x_0 - x_1| \geq |y_0 - y_1|, |z_0 - z_1|$. Apply 2D Bresenham's algorithm to the projection to the xy-plane and to the projection to the xz-plane, and combine the two results.

   (b) (4 points) Consider an algorithm for voxelizing a 3D triangle T with three vertices (x_0, y_0, z_0), (x_1, y_1, z_1), and (x_2, y_2, z_2), where the coordinate values are given as integers. Assuming the triangle T is contained in a plane $L : ax + by + cz + d = 0$, discuss how to apply a sequence of 3D Bresenham's line-drawing algorithm.

   We may assume $|a| \leq |b| \leq |c|$. Now we consider a scanline $y = y_*$ for a 2D triangle T_{xy} with three vertices (x_0, y_0), (x_1, y_1), and (x_2, y_2) in the xy-plane. The line of intersection between the 3D triangle T and the plane $y = y_*$ can be generated by applying 2D Bresenham's algorithm to a line

   $$ax + cz + (by_* + d) = 0.$$

   The end points of this line are taken from the voxelized edges of the triangle T by 3D Bresenham's algorithm.

   Remark: The end points of these 3D lines are approximations to the exact intersection points of the triangle edges and the plane $y = y_*$. Thus the interior voxels thus generated may not be the best approximation to the triangle T.

7. (5 points) Bresenham's algorithm tests the sign of the following integer expression at each step

   $$2[m(x_k + 1) + b] - 2(y_k + 0.5).$$

   Explain how to modify the above expression so that the new expression represents the overlap area of a polygon with the pixel at (x_k, y_k).

   Divide this expression by 2 and add $(1 - m)$. Then we have

   $$[m(x_k + 1) + b] - (y_k + 0.5) = mx_k + b - (y_k + 0.5),$$

   which is the same as the overlap area.