# Chapter 7
# Three-Dimensional Viewing

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# 3D Viewing Pipeline

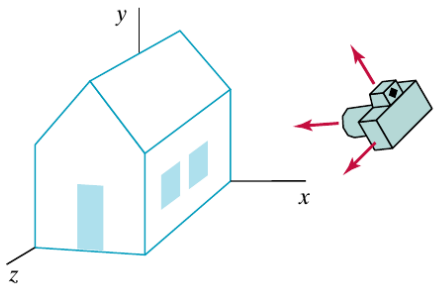



FIGURE 7-10 Photographing a scene involves selection of the camera position and orientation.

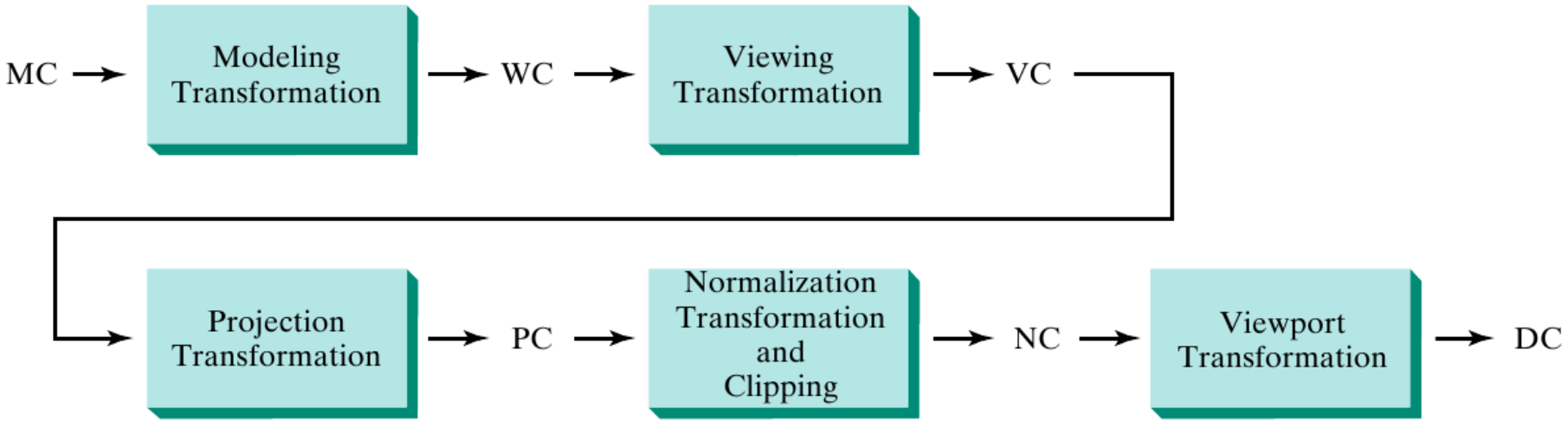



FIGURE 7-11 General three-dimensional transformation pipeline, from modeling coordinates to world coordinates to viewing coordinates to projection coordinates to normalized coordinates and, ultimately, to device coordinates.

# 3D Viewing Parameters

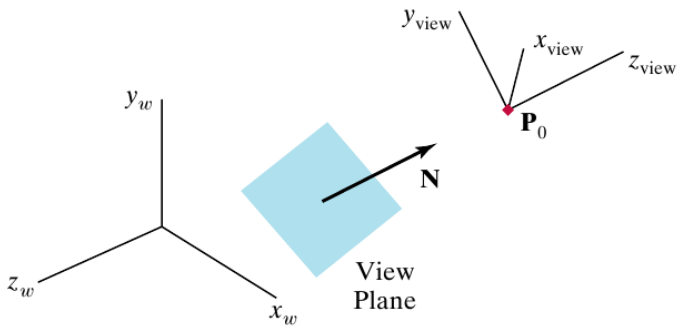



FIGURE 7-13 Orientation of the view plane and view-plane normal vector **N**.

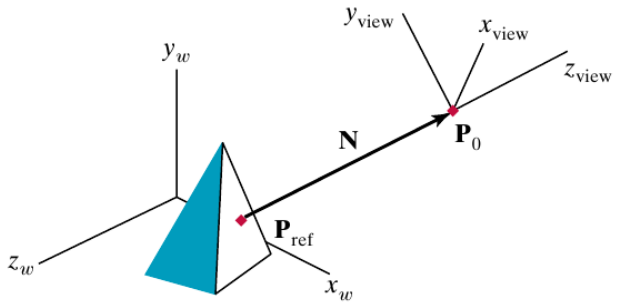



FIGURE 7-15 Specifying the view-plane normal vector **N** as the direction from a selected reference point $\mathbf{P}_{ref}$ to the viewing-coordinate origin $\mathbf{P}_0$.

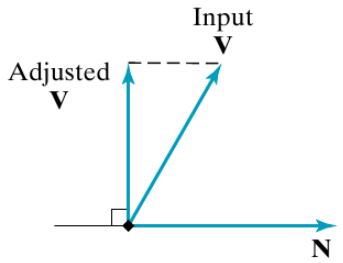



FIGURE 7-16 Adjusting the input direction of the view-up vector **V** to an orientation perpendicular to the view-plane normal vector **N**.

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = (n_x, n_y, n_z)$$

$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{n}}{|\mathbf{V}|} = (u_x, u_y, u_z)$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u} = (v_x, v_y, v_z)$$

$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{n}}{|\mathbf{V} \times \mathbf{n}|} = (u_x, u_y, u_z)$$

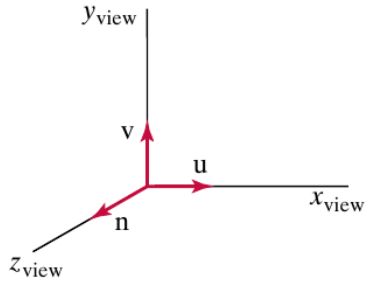



FIGURE 7-17 A right-handed viewing system defined with unit vectors **u**, **v**, and **n**.

# 3D Projections

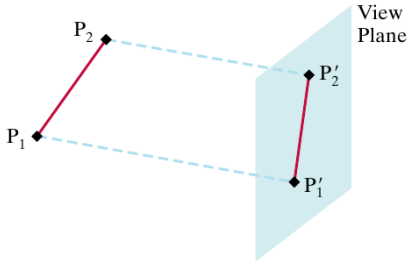



FIGURE 7-22 Parallel projection of a line segment onto a view plane.

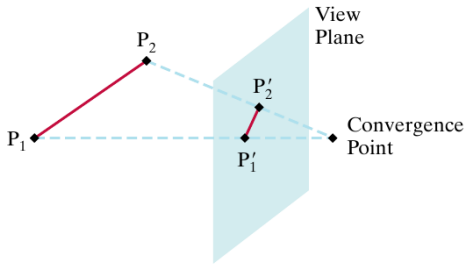



FIGURE 7-23 Perspective projection of a line segment onto a view plane.

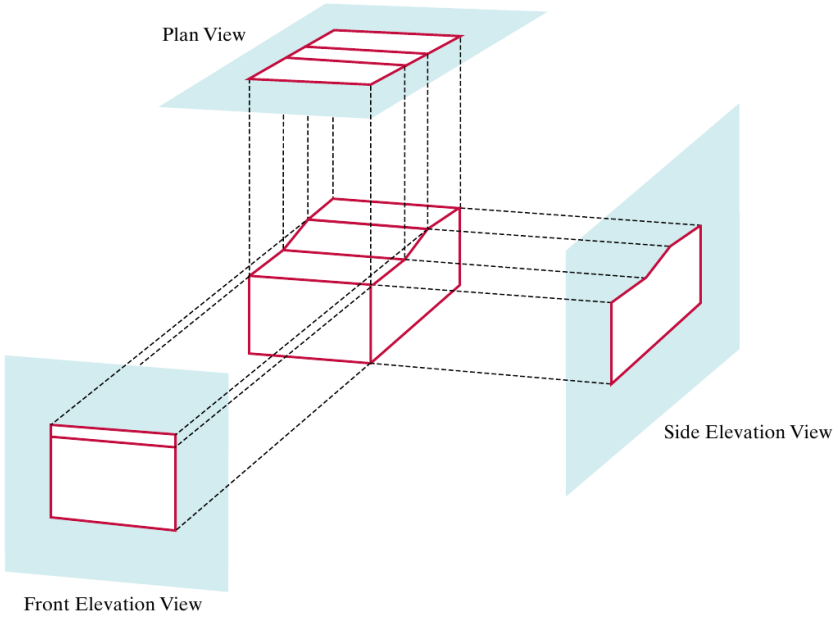
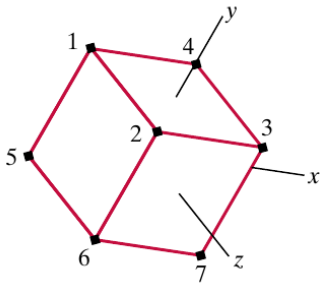



FIGURE 7-24 Orthogonal projections of an object, displaying plan and elevation views.



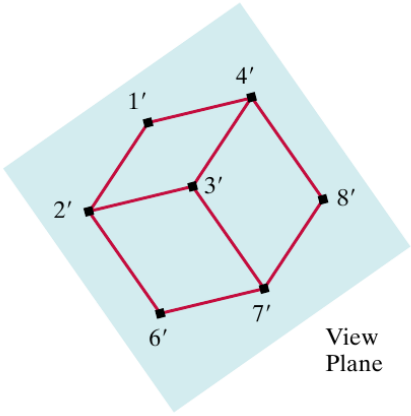




FIGURE 7-25 An isometric projection of a cube.

# Orthogonal Projection

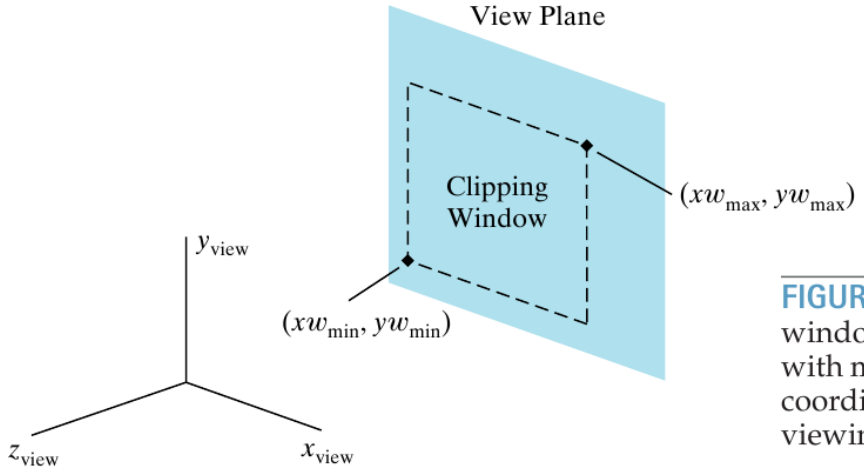



**FIGURE 7-27** A clipping window on the view plane, with minimum and maximum coordinates given in the viewing reference system.

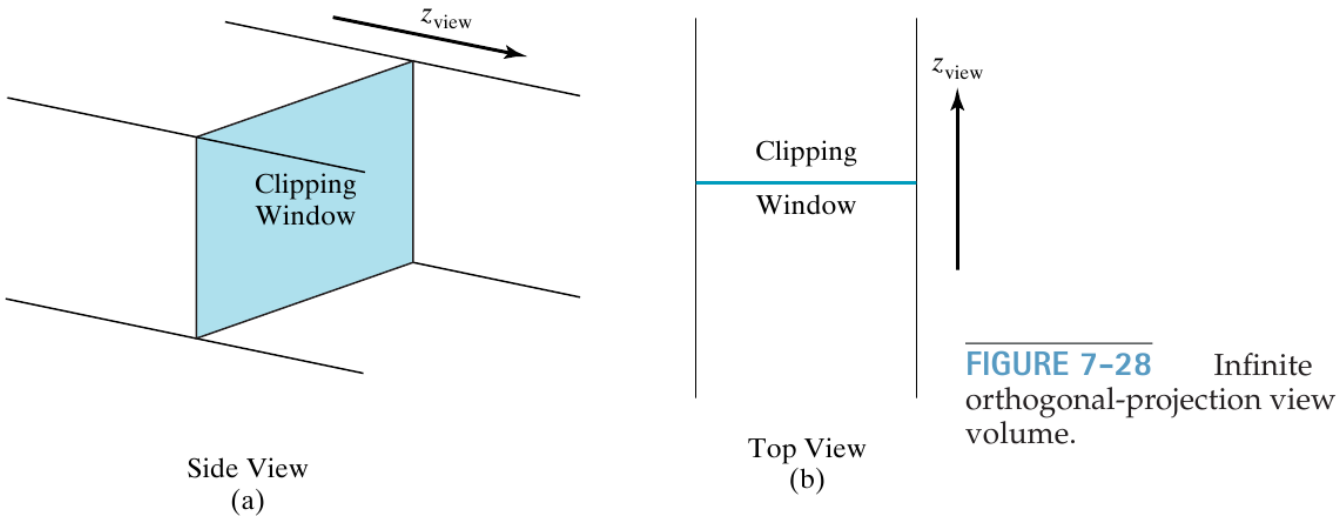



**FIGURE 7-28** Infinite orthogonal-projection view volume.

# Orthogonal Projection

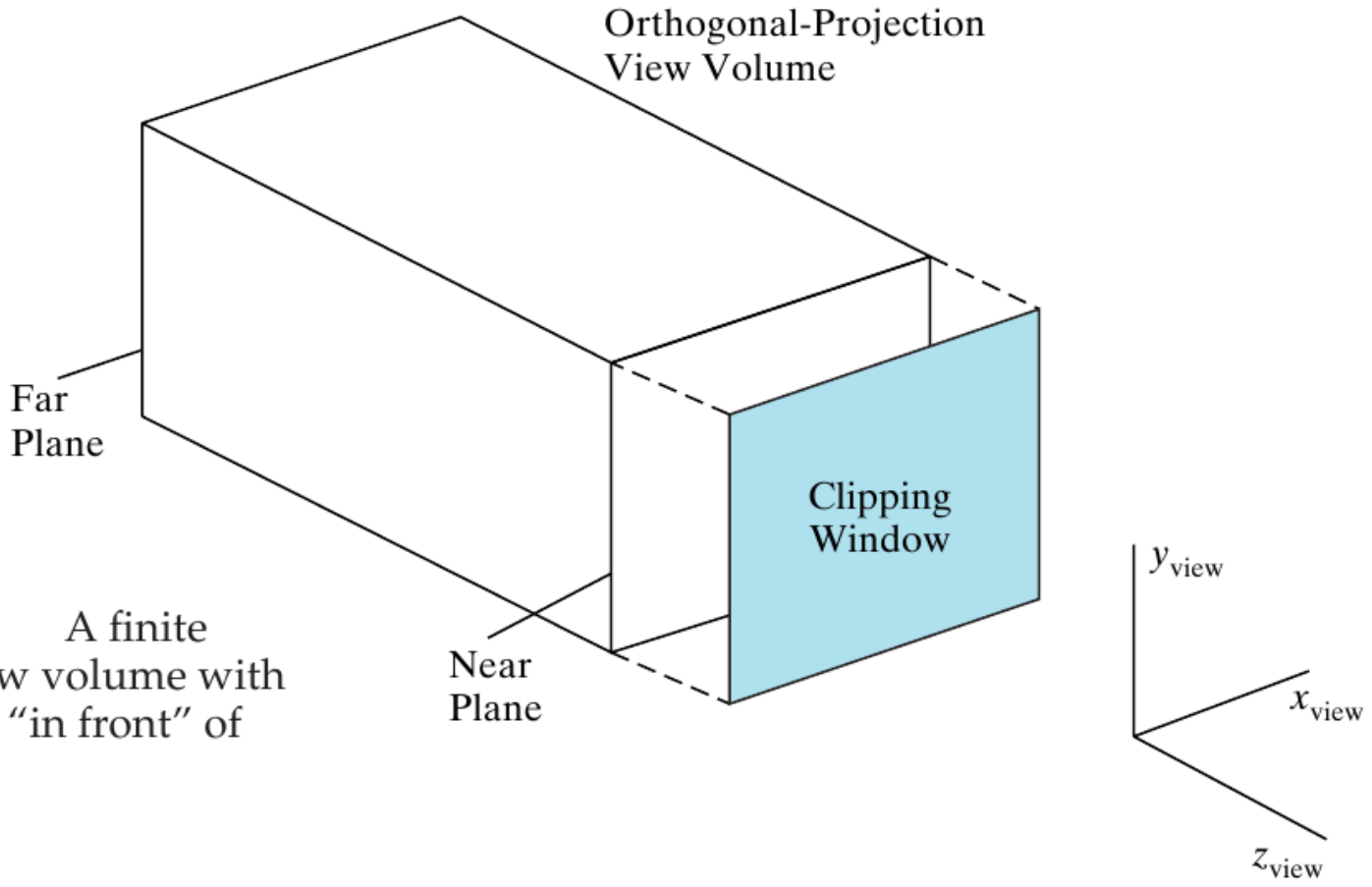



**FIGURE 7-29** A finite orthogonal view volume with the view plane "in front" of the near plane.

# Orthogonal Projection

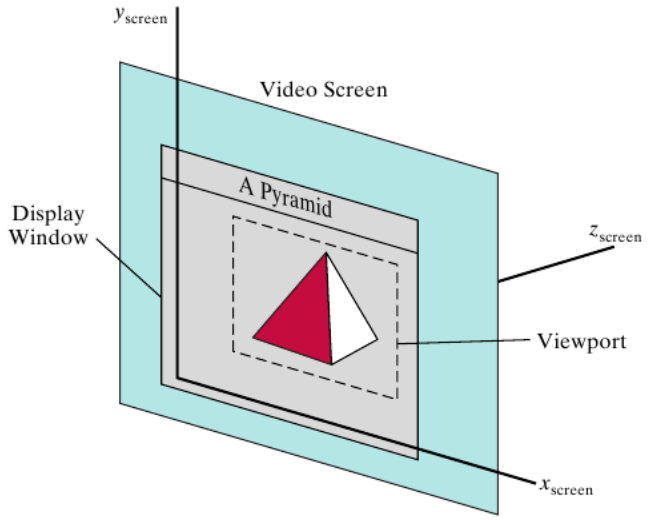



FIGURE 7-30 A left-handed screen-coordinate reference frame.

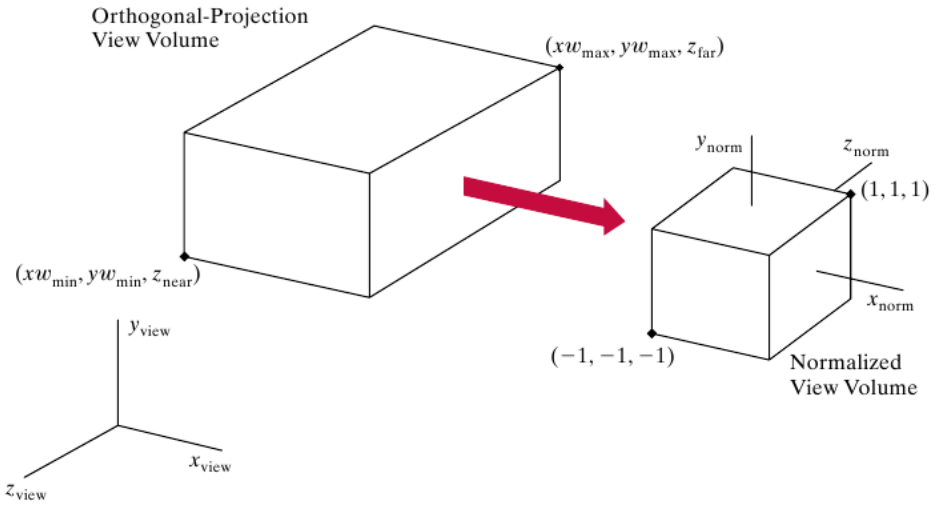



FIGURE 7-31 Normalization transformation from an orthogonal-projection view volume to the symmetric normalization cube within a left-handed reference frame.

# Oblique Parallel Projection

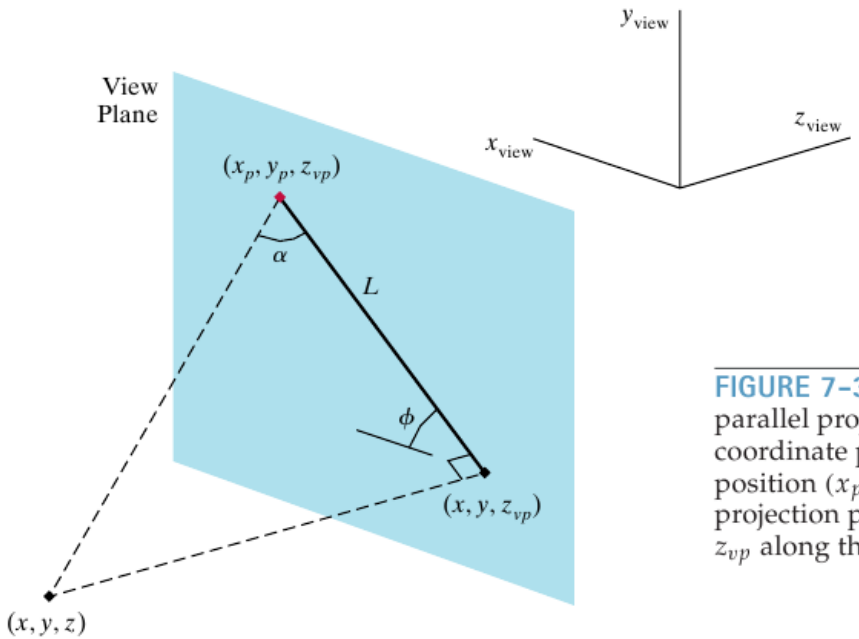
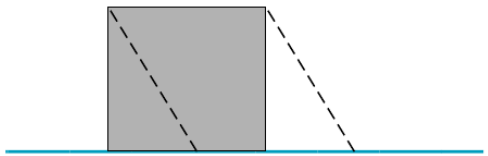


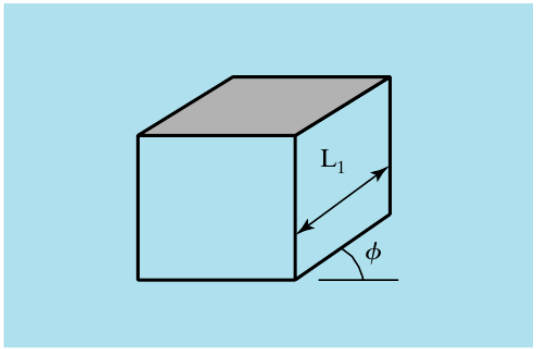

FIGURE 7-33 Oblique parallel projection of coordinate position $(x, y, z)$ to position $(x_p, y_p, z_{vp})$ on a projection plane at position $z_{vp}$ along the $z_{view}$ axis.

$$x_p = x + L\cos\phi$$

$$y_p = y + L\sin\phi$$

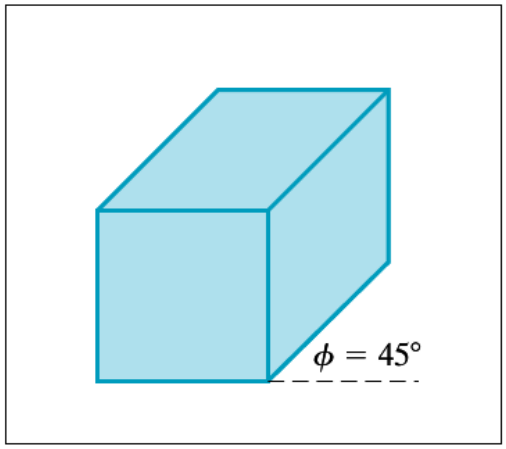



(a)

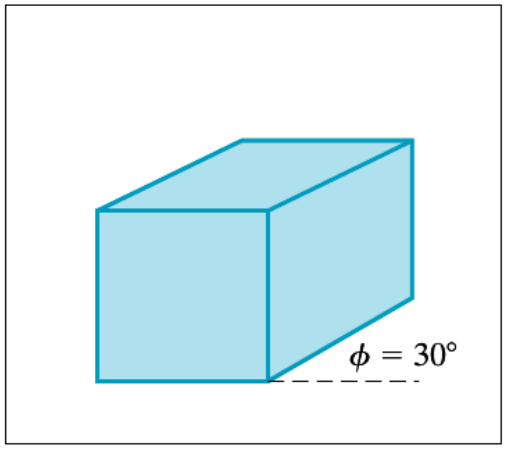



(b)

# Oblique Parallel Projection

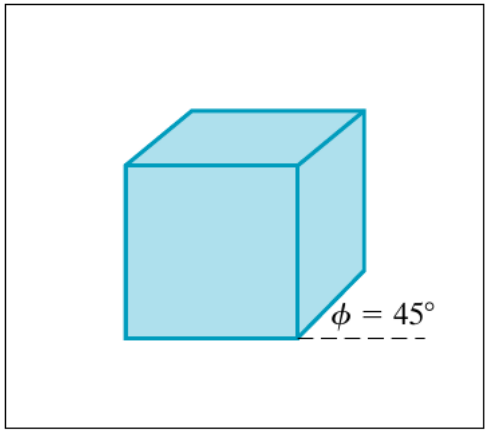



(a)

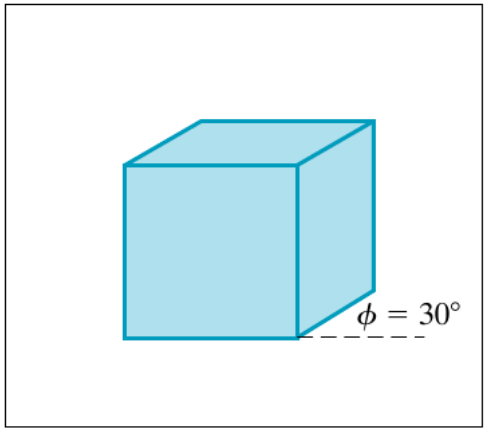



(b)

**FIGURE 7-35** Cavalier projections of a cube onto a view plane for two values of angle $\phi$. The depth of the cube is projected with a length equal to that of the width and height.




(a)




(b)

**FIGURE 7-36** Cabinet projections of a cube onto a view plane for two values of angle $\phi$. The depth is projected with a length that is one half that of the width and height of the cube.

# Oblique Parallel Projection

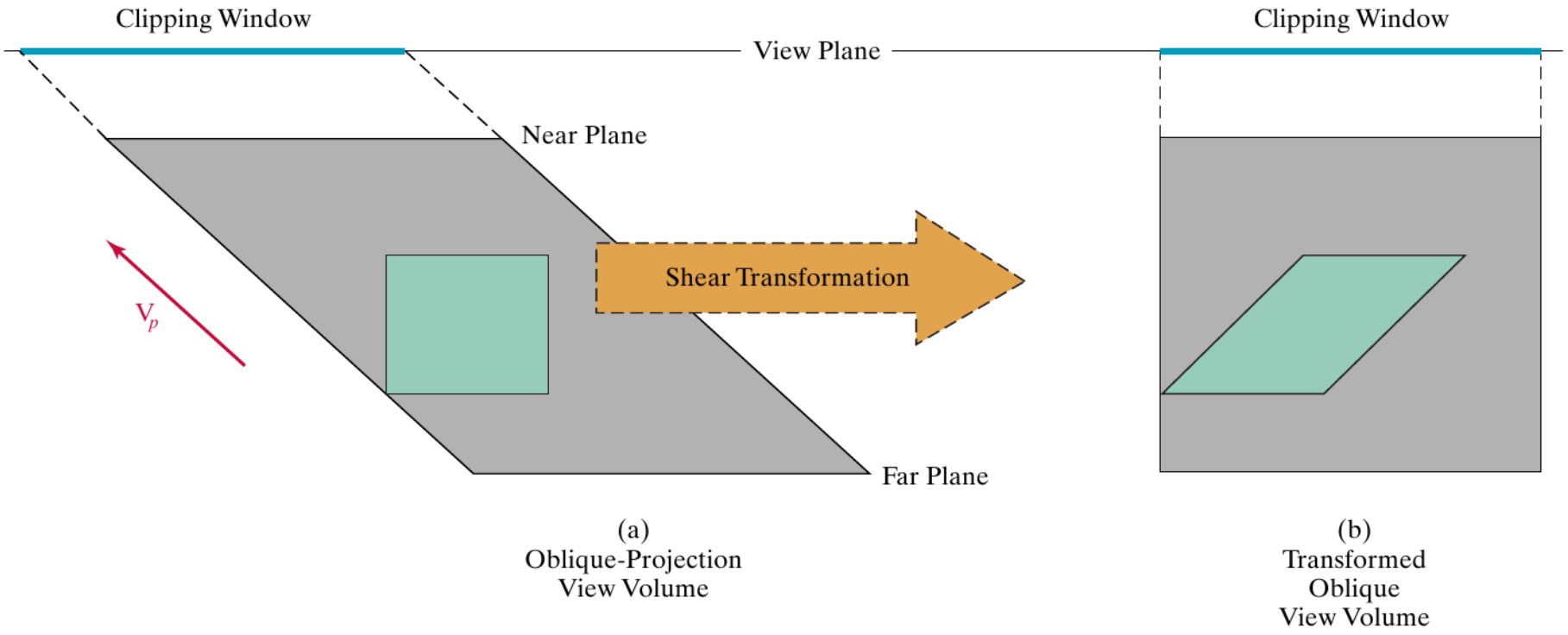



FIGURE 7-39 Top view of an oblique parallel-projection transformation. The oblique view volume is converted into a rectangular parallelepiped, and objects in the view volume, such as the green block, are mapped to orthogonal-projection coordinates.

# 3D Perspective Projection

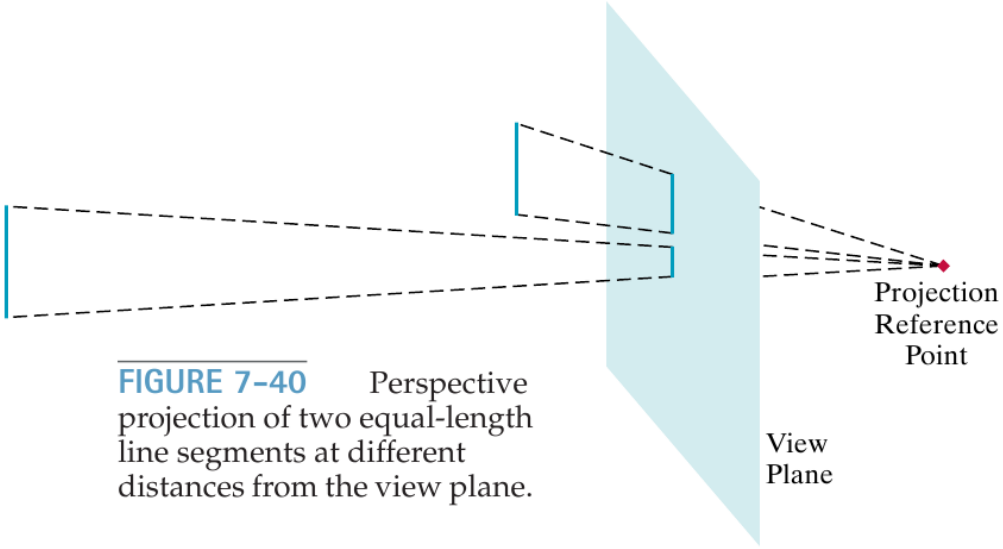



FIGURE 7-40 Perspective projection of two equal-length line segments at different distances from the view plane.

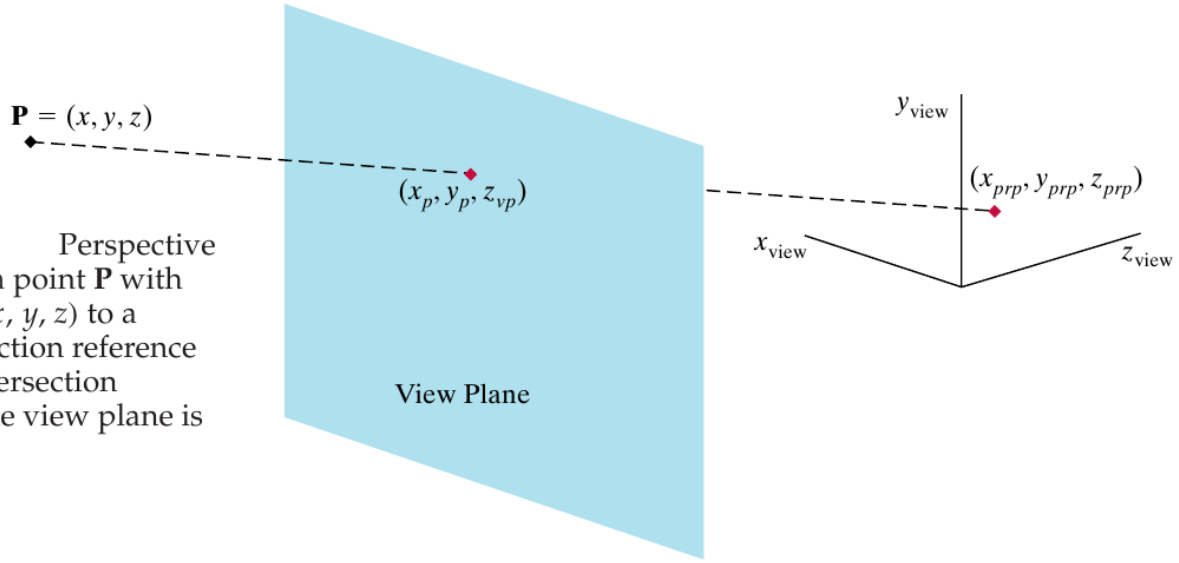



FIGURE 7-41 Perspective projection of a point **P** with coordinates $(x, y, z)$ to a selected projection reference point. The intersection position on the view plane is $(x_p, y_p, z_{vp})$.

# 3D Perspective Projection

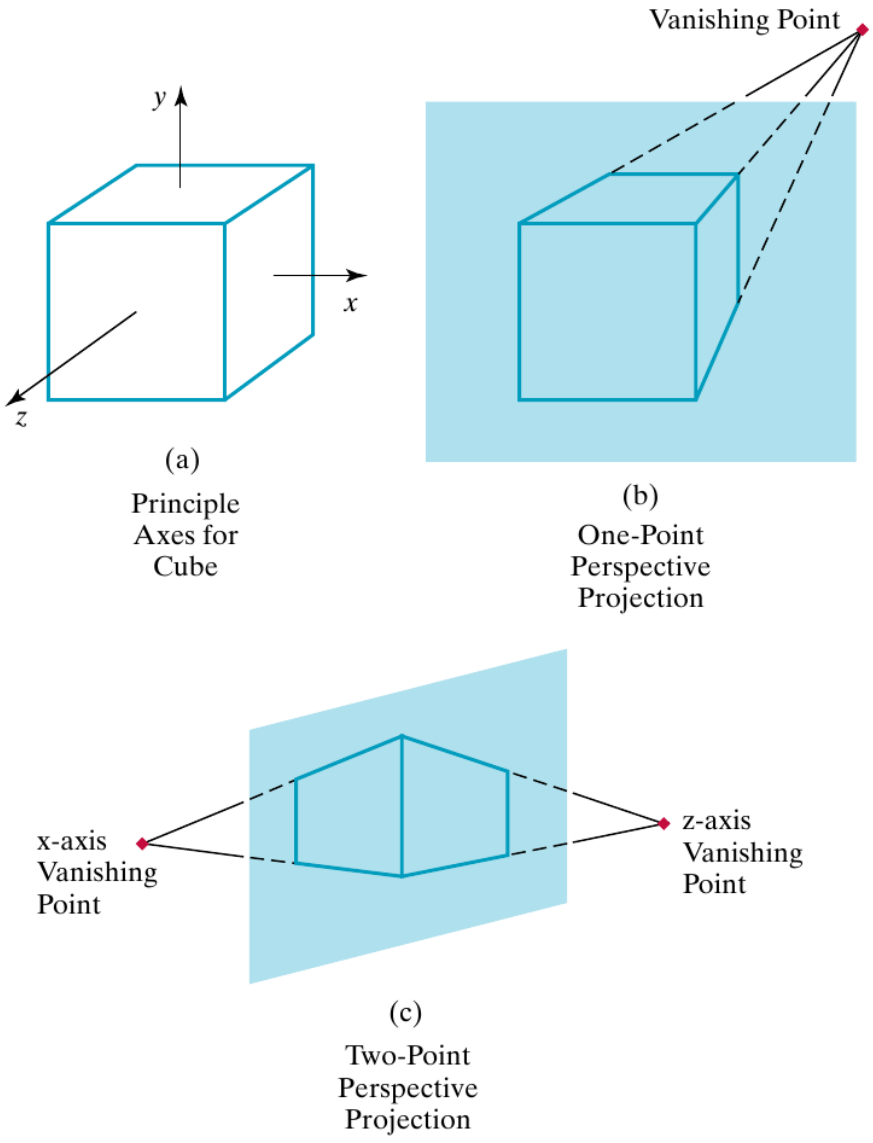



**FIGURE 7-44** Principal vanishing points for perspective-projection views of a cube. When the cube in (a) is projected to a view plane that intersects only the $z$ axis, a single vanishing point in the $z$ direction (b) is generated. When the cube is projected to a view plane that intersects both the $z$ and $x$ axes, two vanishing points (c) are produced.

# 3D Perspective Projection

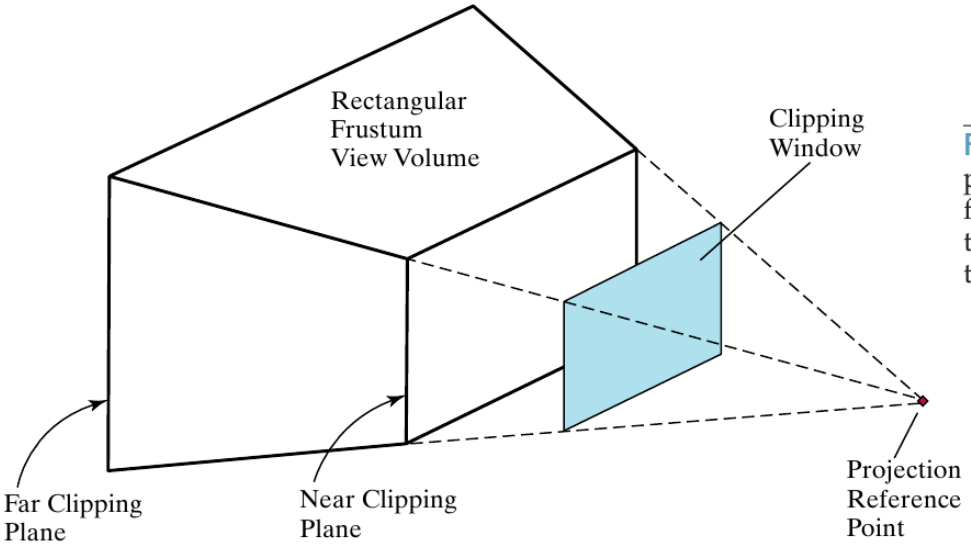



**FIGURE 7-46** A perspective-projection frustum view volume with the view plane "in front" of the near clipping plane.

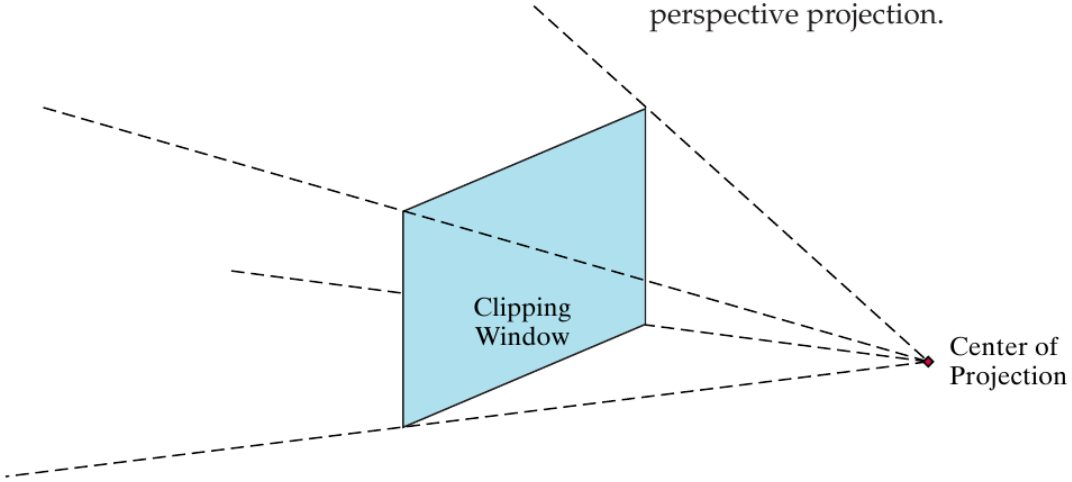



**FIGURE 7-45** An infinite, pyramid view volume for a perspective projection.

# 3D Perspective Projection

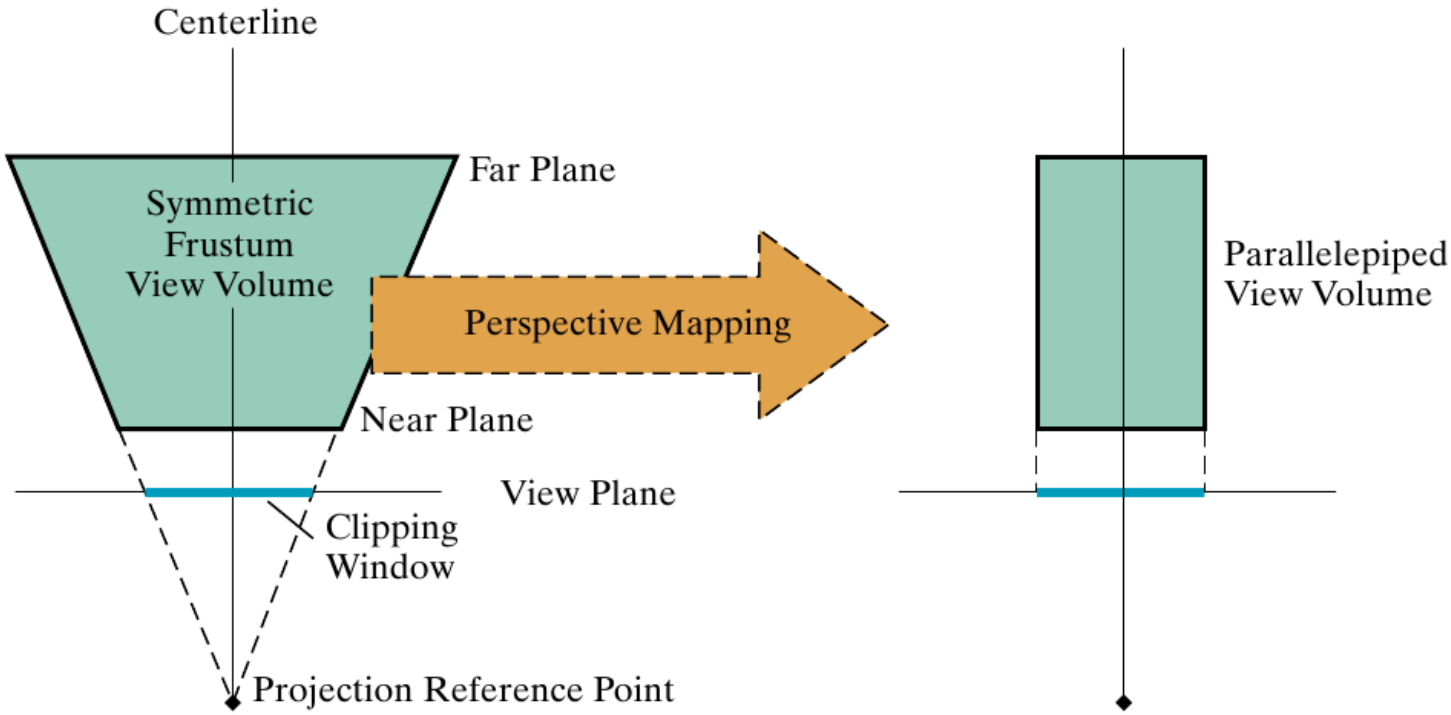



FIGURE 7-51 A symmetric frustum view volume is mapped to an orthogonal parallelepiped by a perspective-projection transformation.

# 3D Perspective Projection

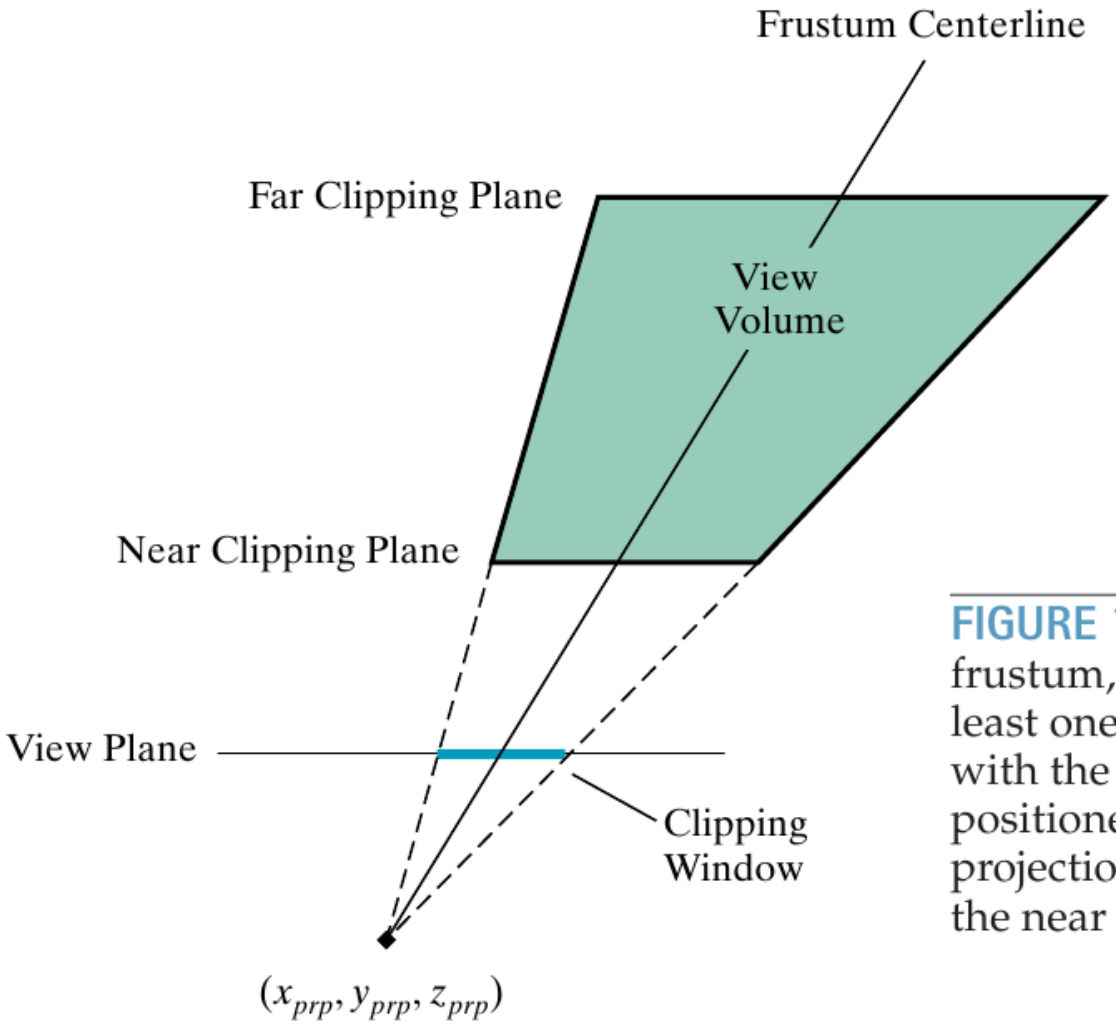



FIGURE 7-52 An oblique frustum, as viewed from at least one side or a top view, with the view plane positioned between the projection reference point and the near clipping plane.

# 3D Clipping Algorithms

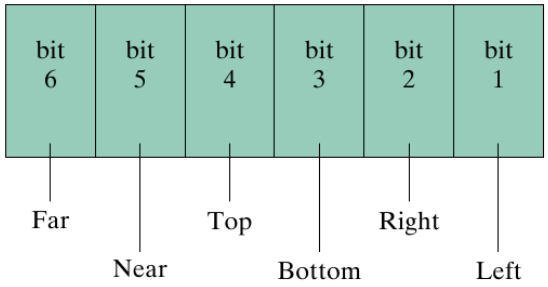
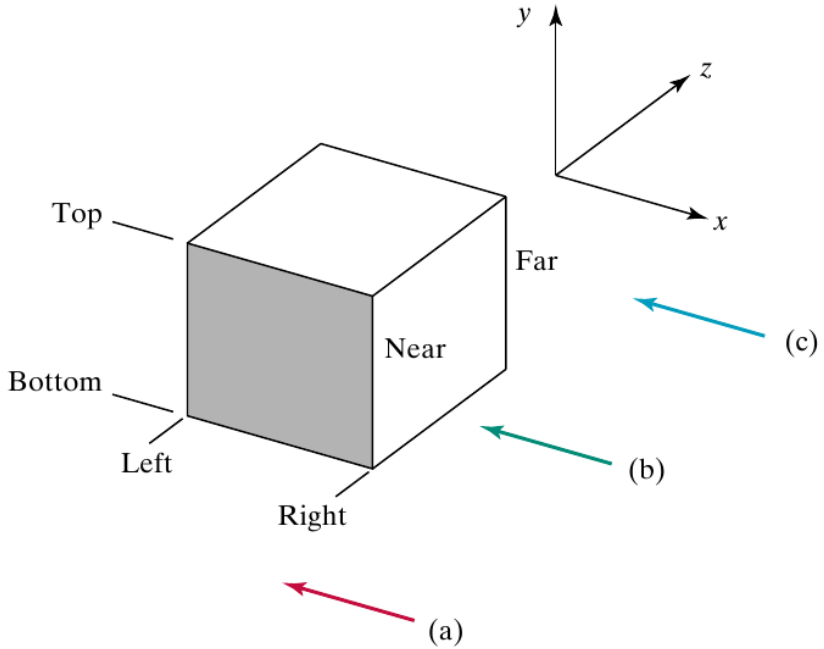



FIGURE 7-56 A possible ordering for the view-volume clipping boundaries corresponding to the region-code bit positions.

FIGURE 7-57 Values for the three-dimensional, six-bit region code that identifies spatial positions relative to the boundaries of a view volume.




| | | |
|---|---|---|
| 011001 | 011000 | 011010 |
| 010001 | 010000 | 010010 |
| 010101 | 010100 | 010110 |

Region Codes
In Front of Near Plane
(a)

| | | |
|---|---|---|
| 001001 | 001000 | 001010 |
| 000001 | 000000 | 000010 |
| 000101 | 000100 | 000110 |

Region Codes
Between Near and Far Planes
(b)

| | | |
|---|---|---|
| 101001 | 101000 | 101010 |
| 100001 | 100000 | 100010 |
| 100101 | 100100 | 100110 |

Region Codes
Behind Far Plane
(c)

# 3D Clipping Algorithms

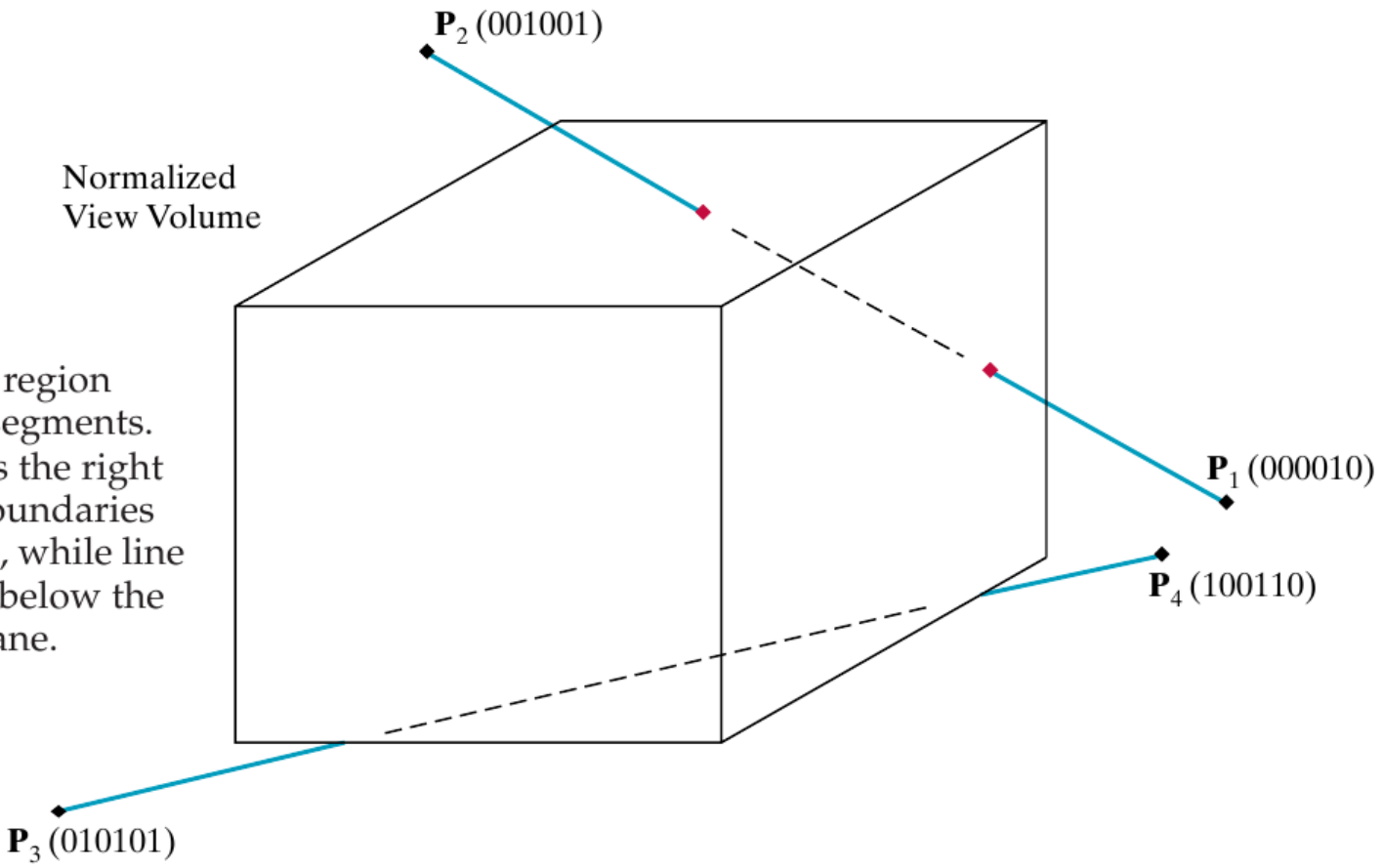



**FIGURE 7-58**
Three-dimensional region codes for two line segments. Line $\overline{\mathbf{P}_1\mathbf{P}_2}$ intersects the right and top clipping boundaries of the view volume, while line $\overline{\mathbf{P}_3\mathbf{P}_4}$ is completely below the bottom clipping plane.

# 3D Polygon Clipping

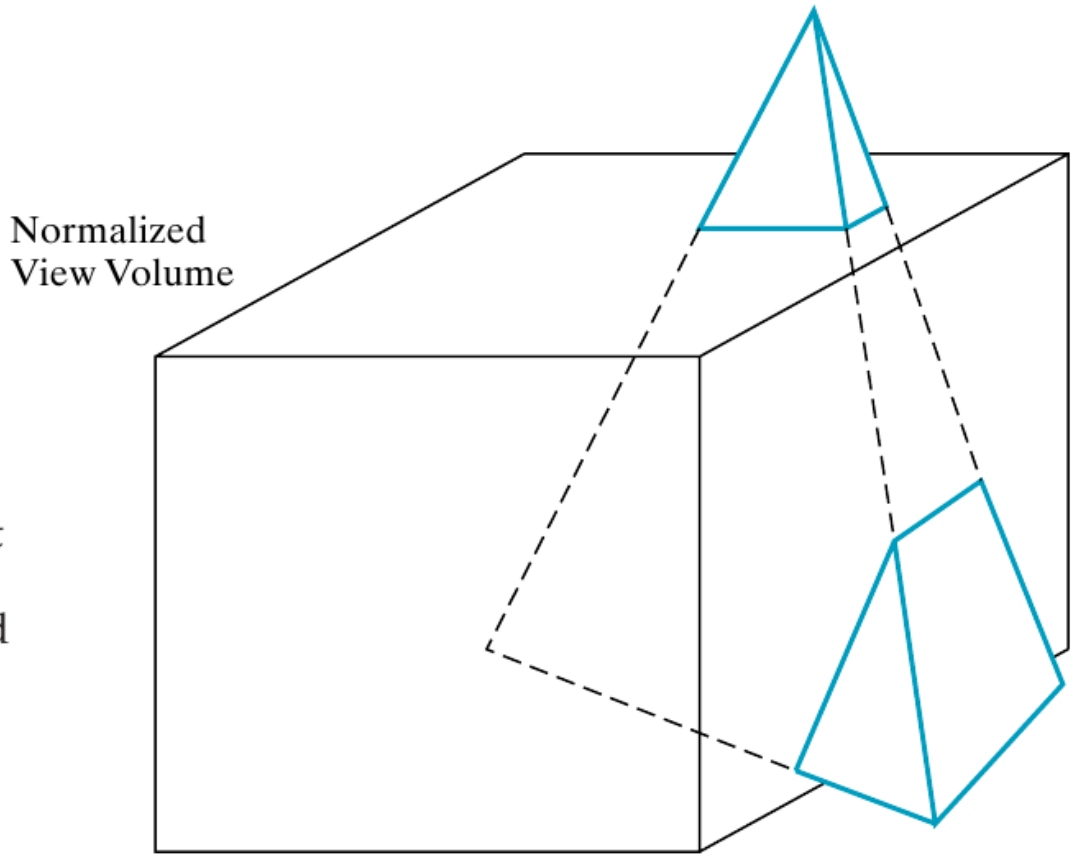



**FIGURE 7-59**
Three-dimensional object clipping. Surface sections that are outside the view-volume clipping planes are eliminated from the object description, and new surface facets may need to be constructed.