Advanced Computer Graphics
(Comp 4190.504)
Midterm Exam: April 14, 2010

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Name: ______________________
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1. (10 points) In Figure 1, what is the problem with the middle sub-figure? How can you fix this problem so as to produce the correct result shown in the rightmost sub-figure?

![polygons and arrows](image)

- The matrix $M$ for transforming points has been used to transform normals.
- $N = (M^{-1})^T = (M^T)^{-1}$ should be used for the transformation of normals.
2. (10 points) Given a unit vector \( \mathbf{u} = \left( \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right) \), find two unit vectors \( \mathbf{v} \) and \( \mathbf{w} \) so that the three vectors \( \mathbf{u}, \mathbf{v}, \mathbf{w} \) form an orthonormal basis.

\[
\mathbf{v} = \left( 0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)
\]

\[
\mathbf{w} = \mathbf{u} \times \mathbf{v}
\]

\[
= \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\
0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}
\end{vmatrix}
\]

\[
= \left( \frac{2\sqrt{2}}{3}, -\frac{\sqrt{2}}{6}, -\frac{\sqrt{2}}{6} \right)
\]
3. (20 points) Consider a rotation \( R_\alpha(180^\circ) \) followed by another rotation \( R_\gamma(90^\circ) \).

(a) (10 points) What is the result of the composite rotation \( R_\gamma(90^\circ)R_\alpha(180^\circ) \)? What are the rotation axis and the rotation angle for the composite rotation?

\[
R_\alpha(180^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad R_\gamma(90^\circ) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}
\]

\[
R_\gamma(90^\circ)R_\alpha(180^\circ) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}
\]

\( w = 0, \quad \cos \theta = 0 \quad \therefore \theta = \pm 90^\circ \)

\( 4x^2 = 0, \quad 4y^2 = -2, \quad 4z^2 = 0, \quad x^2 + y^2 + z^2 = 1 \)

\( \therefore \quad y = 0, \quad z = \pm \frac{1}{2} \)

Rotation about \((\pm \frac{1}{2}, 0, \frac{1}{2})\) by angle \( \pm 180^\circ \)

(b) \( q_1 = (\cos 90^\circ, \sin 90^\circ (1,0,0)) = (0,1,0,0) \)

\( q_2 = (\cos 45^\circ, \sin 45^\circ (0,1,0)) = (\frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}, 0) \)

\( q_3 \cdot q_1 = (0, \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}) \)

\( \cos \theta = 0 \quad \Rightarrow \quad \theta = \pm 90^\circ \)

\( (a,b,c) = (1,0,-1) \)

\( \therefore \quad \text{Rotation about} \ (1,0,-1) \ \text{by angle} \ \pm 180^\circ \).
4. (10 points) What is the point intersection between a line \( ax + by + c = 0 \) and the line connecting two points \((x_0, y_0)\) and \((x_1, y_1)\)?

\[
\begin{align*}
(a, b, c) \times (x_0, y_0, 1) & \times (x_1, y_1, 1) \\
= (a, b, c) \times (y_0 - y_1, x_1 - x_0, x_0y_1 - x_1y_0) \\
= (b(x_0y_1 - x_1y_0) - c(x_1 - x_0), c(y_0 - y_1) - a(x_0y_1 - x_1y_0), \\
& \quad a(x_1 - x_0) - b(y_0 - y_1))
\end{align*}
\]

The point of intersection is

\[
\left( \frac{b(x_0y_1 - x_1y_0) - c(x_1 - x_0)}{a(x_1 - x_0) - b(y_0 - y_1)}, \frac{c(y_0 - y_1) - a(x_0y_1 - x_1y_0)}{a(x_1 - x_0) - b(y_0 - y_1)} \right)
\]
5. (20 points)

(a) (10 points) What are the main differences between the space partitioning trees and the BVH trees?

(b) (10 points) Discuss how to proceed the hierarchical view frustum culling using spatial data structures?

(a) ① BSP trees and octrees subdivide the whole space without overlap (an exception is loose octrees). On the other hand, a BVH encloses the regions of the space surrounding geometric objects, and thus the BVH may not enclose the whole space.

② BSP trees and octrees are useful in occlusion ordering/culling.

③ BVH trees are useful in updating their structures in dynamic scenes (loose octrees are an exception).

(b) ① BVH: at each step, the BV of a node is

- outside of the frustum: cull the subtree
- inside: the contents are sent for rendering
- intersecting: continue the recursion to subtrees

② BSP or Octrees: at each step, the splitting plane

- intersects the frustum: both branches are traversed.
- the frustum is fully: the other side is culled in one side and continue the traversal on the side of full containment.
6. (10 points) What are the main differences among the triangle strips, fans, and meshes?

0. Any triangle fan can be represented as a triangle strip (with many swaps), but not vice versa.

0. Strips and fans allow some data sharing, but meshes allow full sharing.

0. For meshes, it is important to send the indices of vertices in an order with good spatial coherence. Thus, a recursive algorithm can be used to split the mesh into submeshes of approximately equal size.

0. Fans are often used when polygons are triangulated.

0. Strips can be generated by an algorithm for finding the longest paths in the dual graph of a mesh.
7. (20 points) Given a quadrilateral with four corners \( p_{00}, p_{01}, p_{10}, p_{11} \). What is the maximum deviation of the quadrilateral from its best fitting plane? Justify your answer.

\[
(1-u)(1-v) p_{00} + (1-u)v p_{01} + u(1-v) p_{10} + uv p_{11}
\]

Let \( C = \frac{1}{4} (p_{00} + p_{01} + p_{10} + p_{11}) \).

The best fitting plane contains \( C \) and is parallel to

\[
\frac{1}{2} (p_{11} + p_{10} - p_{01} - p_{00}) \quad \text{and} \quad \frac{1}{2} (p_{11} + p_{01} - p_{10} - p_{00})
\]

\[
\frac{1}{2} (p_{00} + p_{01}) \quad \frac{1}{2} (p_{10} + p_{11})
\]

The plane is also parallel to:

\[
p_{11} - p_{00} \quad \text{and} \quad p_{10} - p_{01}
\]

Let \( m = \frac{(p_{11} - p_{00}) \times (p_{10} - p_{01})}{\lVert (p_{11} - p_{00}) \times (p_{10} - p_{01}) \rVert} \).

The maximum deviation is

\[
\frac{1}{4} \lVert <p_{00} + p_{11} - p_{01} - p_{10}, m> \rVert
\]