Quiz #3 (CSE 4190.313)

Wednesday, May 12, 2010

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1. (10 points) B is a 3×3 matrix with three eigenvalues $0, 1, 2$. This information is enough to find three of these:

   (a) the rank of B, $=2$

   (b) the determinant of $B^T B$, $=0$

   (c) the eigenvalues of $B^T B$, and Not enough information

   (d) the determinant of $(B+I)^{-1}$. $= 1/6$

(a) $B x_1 = 0 \cdot x_1$, $B x_2 = x_2$, $B x_3 = 2 x_3$

rank $B \leq 2$ ($\because$ x_1 is in the nullspace of B)

rank $B \geq 2$ ($\because$ x_2 and x_3 are two lin. indep. vectors in the column space of B)

$\therefore$ rank $B = 2$

(b) $\det(B^T B) = (\det B^T)(\det B) = (\det B)^2 = 0$

(c) B may be non-symmetric.

(d) $(B+I)x_1 = x_1$, $(B+I)x_2 = 2x_2$, $(B+I)x_3 = 3x_3$

$\det(B+I) = 6$

$\therefore \det(B+I)^{-1} = 1/6$

2. (10 points) What is the limit as $k \to \infty$ of

$$\begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 0.4-\lambda & 0.3 \\ 0.6 & 0.7-\lambda \end{vmatrix} = \lambda^2 - 1.1\lambda + 0.1 = 0$$

$$\therefore \lambda = 1, 0.1$$

$$\lambda_1 = 1, \quad x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \lambda_2 = 0.1, \quad x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A^k = S\Lambda^k S^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1^k & \\ & (0.1)^k \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\lim_{k\to\infty} A^k = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\lim_{k\to\infty} A^k \begin{bmatrix} a \\ b \end{bmatrix} = (a+b)\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

3. (10 points)

(a) (5 points) What matrix M changes the basis $V_1 = (1,1), V_2 = (1,4)$ to the basis $\mathbf{v}_1 = (2,5), \mathbf{v}_2 = (1,4)$?

(b) (5 points) For the same two bases, express the vector $(3,9)$ as a combination $c_1V_1 + c_2V_2$ and also as $d_1\mathbf{v}_1 + d_2\mathbf{v}_2$. Check numerically that M connects $\mathbf{c} = (c_1, c_2)^T$ to $\mathbf{d} = (d_1, d_2)^T$: $M\mathbf{c} = \mathbf{d}$.

(a) $\left.\begin{array}{l} V_1 = \mathbf{v}_1 - \mathbf{v}_2 \\ V_2 = \mathbf{v}_2 \end{array}\right) \Rightarrow M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

(b) $(3,9) = V_1 + 2V_2 \Rightarrow \mathbf{c} = (1,2)^T$

$(3,9) = \mathbf{v}_1 + \mathbf{v}_2 \Rightarrow \mathbf{d} = (1,1)^T$

$M\mathbf{c} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \mathbf{d}$!