Quiz #2 (CSE 4190.313)

Wednesday, April 7, 2010

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1. (8 points) Without computing A, find bases for the four fundamental subspaces:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

① $C(A)$: $\{(1,1,1), (0,1,1), (0,0,1)\}$

② $N(A)$: $\{(0,1,-2,1)\}$

③ $C(A^T)$: $\{(1,2,3,4), (0,1,2,3), (0,0,1,2)\}$

④ $N(A^T)$: $\phi \rightarrow$ basis 자체는 empty

$\quad\quad \parallel$

$\{(0,0,0)\}$

2. (4 points) The space of all 2×2 matrices has the four basis "vectors"

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, V_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, V_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

For the linear transformation of transposing, find its matrix representation A with respect to this basis.

$A(V_1) = V_1$, $A(V_2) = V_3$, $A(V_3) = V_2$, $A(V_4) = V_4$

+2

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ +2

3. (5 points) What are the special solutions to $R\mathbf{x} = \mathbf{0}$?

$$R = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = (-1, -2, 1, 0)$ +3

$x_2 = (-2, -3, 0, 1)$ +2

4. (4 points) For which numbers c and d, does the following matrix A have rank 2?

$$A = \begin{bmatrix} 1 & 2 & 2 & 0 & 2 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}$$

$c \neq 0$ implies $\text{rank}(A) = 3$ (+2)

Thus, $c = 0$ and $d = 2$ (+2)

$\because$ Otherwise, $\text{rank}(A) = 3$ $\lrcorner$

5. (4 points) True or false (give a good reason)?

(a) If the columns of A are linearly independent, then $A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $\mathbf{b}$.

(b) A 5×7 matrix never has linearly independent columns.

(a) False : Counter Example

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbb{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \mathbb{b} \notin C(A)$$

There is no $\mathbb{x}$ such that $A\mathbb{x} = \mathbb{b}$.

(b) True

$\because$ Seven vectors in $\mathbb{R}^5$ are always linearly dependent. $\lrcorner$