Quiz #3 (CSE 4190.313)

Monday, April 11, 2011

Name: ____________________ **E-mail:** ____________________

Dept: ____________________ **ID No:** ____________________

1. (4 points) Suppose all vectors $\mathbf{x}$ in the unit square $[0,1]\times[0,1] = \{(x_1, x_2) \mid 0 \le x_1, x_2 \le 1\}$ are transformed to $A\mathbf{x}$, where A is a 2 by 2 matrix.

   (a) What is the shape of the transformed region (all $A\mathbf{x}$)?

   (b) For which matrices A is that region a square?

   (c) For which A is it a line?

   (d) For which A is the new area still 1?

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(a) Parallelogram with four corners:

$(0,0)$, (a,c), (b,d), $(a+b, c+d)$

(b) $ab+cd=0$, $a^2+c^2=b^2+d^2 \neq 0$

(c) $ad-bc=0$, but $A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(d) $ad-bc=\pm 1$

2. (3 points) Find the dimensions of these vector spaces:

(a) The space of all vectors in R^3 whose components add to zero. 2 dim.

(b) The nullspace of the 3 by 3 identity matrix. 0 dim.

(c) The space of all 3 by 3 matrices. 9 dim.

(a) $\{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$

$(x,y,z) = (x, y, -x-y) = x(1,0,-1) + y(0,1,-1)$

(b) $N(I) = \{0\}$

(c) Basis : $\left\{ \begin{bmatrix} 1&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix}, \cdots, \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&1 \end{bmatrix} \right\}$

3. (4 points) True or false (give a good reason or a counterexample)?

(a) The vectors **b** that are not in the column space $C(A)$ form a subspace.

(b) If $C(A)$ contains only the zero vector, then A is the zero matrix.

(c) The column space of $2A$ equals to the column space of A.

(d) The column space of $A - I$ equals the column space of A.

(a) False. ($\because$ $0 \in \mathbb{R}^n \setminus C(A)$)

(b) True. $\ulcorner \because$ If A were not the zero matrix,

A has a non-zero column vector.

$\Rightarrow C(A) \neq \{0\}$ # $\lrcorner$

(c) True. $\ulcorner \because$ Let $a_1, \cdots, a_n$: columns of A,

$\Rightarrow c_1(2a_1) + \cdots + c_n(2a_n)$

$= (2c_1)a_1 + \cdots + (2c_n)a_n$ $\lrcorner$

(d) False. $\ulcorner \because$ When $A = I$, $C(A-I) = C(0) = \{0\}$

$C(A) = C(I) = \mathbb{R}^n$ $\lrcorner$

4. (9 points) Given

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix},$$

(a) (3 points) Find the special solutions of $A\mathbf{x} = \mathbf{0}$.

(b) (3 points) Find the column space and nullspace of A.

(c) (3 points) Find a particular solution to $A\mathbf{x} = \mathbf{b}$.

(a)
$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

$$\to \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R$$

Special Solutions: $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

(b) Basis of $C(A)$: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \right\}$

Basis of $N(A) = \{$ 〃 , 〃 $\}$

(c)
$$\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \to \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \to \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$$

$$\mathbf{x}_p = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$