Quiz #4 (CSE 4190.313)

Monday, April 25, 2011

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1. (5 points) If $W = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, find the W-inner product of $\mathbf{x} = (2,3)$ and $\mathbf{y} = (1,1)$, and the W-length of $\mathbf{x}$. What line of vectors is W-perpendicular to $\mathbf{y}$?

① $(x, y)_W = (Wy)^T(Wx)$

$= y^T W^T W x$

$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 11$

+2

② $\|x\|_W = \|Wx\| = \|(4,3)\| = 5$

+1

③ $((x_1, x_2), (1,1))_W = 0$

$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$4x_1 + x_2 = 0$

+2

2. (5 points) Apply Gram-Schmidt to

$$\mathbf{a} = (1, -1, 0, 0), \quad \mathbf{b} = (0, 1, -1, 0), \quad \mathbf{c} = (0, 0, 1, -1),$$

to find orthogonal vectors A, B, C.

$$A = \mathbf{a} = (1, -1, 0, 0) \quad +1$$

$$B = \mathbf{b} - \frac{\langle A, \mathbf{b} \rangle}{\langle A, A \rangle} A$$

$$= (0,1,-1,0) - \frac{(-1)}{2}(1,-1,0,0)$$

$$= (0,1,-1,0) + \left(\frac{1}{2}, -\frac{1}{2}, 0, 0\right)$$

$$= \left(\frac{1}{2}, \frac{1}{2}, -1, 0\right) \quad +2$$

$$C = \mathbf{c} - \frac{\cancel{\langle A, \mathbf{c} \rangle} \to 0}{\langle A, A \rangle} A - \frac{\langle B, \mathbf{c} \rangle}{\langle B, B \rangle} B$$

$$= (0,0,1,-1) - 0 - \frac{(-1)}{\frac{1}{4} + \frac{1}{4} + 1} \cdot \left(\frac{1}{2}, \frac{1}{2}, -1, 0\right)$$

$$= (0,0,1,-1) + \frac{2}{3}\left(\frac{1}{2}, \frac{1}{2}, -1, 0\right)$$

$$= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right) \quad +2$$

3. (10 points) Construct a matrix with the required property or say why that is impossible.

(a) Column space contains $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, null space contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(b) Row space contains $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, null space contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(c) $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a solution and $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(d) Every row is orthogonal to every column (A is not the zero matrix).

(e) The columns add up to a column of 0s, the rows add up to a row of 1s.

(a) $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -3 & 1 \\ -3 & 5 & -2 \end{bmatrix}$

(b) Impossible

$\because$ (1,1,1) cannot be orthogonal to (2,−3,5)

(c) Impossible

[(1,1,1) is in the column space of A
(1,0,0) is in the left nullspace of A

But, they are not orthogonal #

$\because$ The first row of A should be the zero vector.

$\Rightarrow A\mathbf{x} = \begin{bmatrix} 0 \\ * \\ * \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ #

(d) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

(e) Impossible

[$\because$ (1,1,⋯) is in the nullspace of A
at the same time, it is in the rowspace of A #

$\because$ $\sum_{j=1}^{n} a_{ij} = 0$ for all $i \Rightarrow \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} = 0$

$\sum_{i=1}^{m} a_{ij} = 1$ for all $j \Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij} = n$

$\therefore 0 = n$ #