Quiz #1 (CSE 4190.313)

Wednesday, March 9, 2011

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1. (4 points) By trial and error find examples of 2 by 2 matrices such that

   (a) $A^2 = -I$, A having only real entries.

   (b) $B^2 = 0$, although $B \neq 0$.

   (c) $CD = -DC$, not allowing the case $CD = 0$.

   (d) $EF = 0$, although no entries of E or F are zero

(a) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

(b) $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B \neq 0$

(c) $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$CD = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $DC = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$\therefore CD = -DC$

(d) $E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$EF = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(+1)

2. (3 points) Normally 4 "planes" in four-dimensional space meet at a point. Normally 4 column vectors in four-dimensional space can combine to produce **b**. What combination of $(1,0,0,0)$, $(1,1,0,0)$, $(1,1,1,0)$, $(1,1,1,1)$ produces $\mathbf{b} = (3,3,3,2)$? What 4 equations for x, y, z, t are you solving?

$$x\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + y\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + z\begin{bmatrix}1\\1\\1\\0\end{bmatrix} + t\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\3\\3\\2\end{bmatrix}$$

$$\begin{cases} x+y+z+t=3 \\ y+z+t=3 \\ z+t=3 \\ t=2 \end{cases} \Rightarrow \begin{cases} t=2 \\ z=1 \\ y=0 \\ x=0 \end{cases}$$

(+1) (+1)

3. (3 points) For which three number a will elimination fail to give three pivots?

$$\begin{aligned} ax+2y+3z &= b_1 \\ ax+ay+4z &= b_2 \\ ax+ay+az &= b_3. \end{aligned}$$

$$\begin{cases} ax+2y+3z=b_1 \\ (a-2)y+z=b_2-b_1 \\ (a-2)y+(a-3)z=b_3-b_1 \end{cases}$$

$$\Rightarrow \begin{cases} ax+2y+3z=b_1 \\ (a-2)y+z=b_2-b_1 \\ (a-4)z=b_3-b_2 \end{cases}$$

$\therefore\ a=0,\ 2,$ or 4