

## Quiz #2 (CSE4190.410)

September 27, 2010 (Monday)

Name: \_\_\_\_\_ Dept: \_\_\_\_\_ ID No: \_\_\_\_\_

1. (10 points) Consider a tetrahedron with four vertices  $A(0,0,0)$ ,  $B(1,0,0)$ ,  $C(0,1,0)$ , and  $D(0,0,1)$ . Apply a perspective projection to this tetrahedron onto the viewplane  $3x - 4y + 5z = 0$  from the viewpoint  $(1, -2, 3)$  and compute the four projected points  $A'$ ,  $B'$ ,  $C'$ , and  $D'$ .

$$\hat{m} = (3, -4, 5, 0), \quad \hat{v} = (1, -2, 3, 1)$$

$$\hat{x}' = \hat{m} \times (\hat{x} \times \hat{v}) = \langle \hat{m}, \hat{v} \rangle \hat{x} - \hat{v} \langle \hat{m}, \hat{x} \rangle$$

$$= 26 \hat{x} - \begin{bmatrix} 3 & -4 & 5 & 0 \\ -6 & 8 & -10 & 0 \\ 9 & -12 & 15 & 0 \\ 3 & -4 & 5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 4 & -5 & 0 \\ 6 & 18 & 10 & 0 \\ -9 & 12 & 11 & 0 \\ -3 & 4 & -5 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{A}' & \hat{B}' & \hat{C}' & \hat{D}' \end{bmatrix} = \begin{bmatrix} 0 & 23 & 4 & -5 \\ 0 & 6 & 18 & 10 \\ 0 & -9 & 12 & 11 \\ 26 & 23 & 30 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2/15 & -5/21 \\ 0 & 6/23 & 3/5 & 10/21 \\ 0 & -9/23 & 2/5 & 11/21 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

2. (5 points) Consider two parallel planes:

$$\Pi_1 : 3x - 4y + 5z + 6 = 0,$$

$$\Pi_2 : 3x - 4y + 5z - 6 = 0.$$

What is the affine transformation from  $R^3$  to  $R^1$  that sends  $\Pi_1$  to 0 and  $\Pi_2$  to 1?

$$\begin{bmatrix} 3 & -4 & 5 & 6 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

3. (5 points) Write down an algebraic expression for testing whether the following four planes share a common intersection point:

$$\Pi_i : a_i x + b_i y + c_i z + d_i = 0, \quad (i = 1, 2, 3, 4).$$

$$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0$$