Quiz #1 (CSE 4190.313)

Monday, March 22, 2010

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (5 points) Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \qquad +1$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix} \qquad +2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 9 & -3 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 9 & -3 & 1 & 0 \\ -12 & 4 & -2 & 1 \end{bmatrix} \qquad +2$$

2. (4 points) How many steps does elimination use in solving 10 systems with the same 60 by 60 coefficient matrix A?

$A = LU$ takes $\frac{1}{3} \times 60^3 = 20 \times 60^2$ steps (+1)

$\left.\begin{array}{l} L\mathbf{c}_i = \mathbf{b}_i \\ U\mathbf{x}_i = \mathbf{c}_i \end{array}\right]$ takes $2 \times \frac{1}{2} \times 60^2 = 60^2$ steps for $i = 1, \cdots, 10$. (+2)

$\therefore\ 20 \times 60^2 + 10 \times 60^2 = 30 \times 60^2 = 108{,}000$ steps (+1)

3. (4 points) True or false? Give a specific counterexample when false.

(a) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.

(b) If rows 1 and 3 of A are the same, so are rows 1 and 3 of AB.

(a) $AB = A[\mathbf{b}_1 \cdots \mathbf{b}_n] = [A\mathbf{b}_1\ A\mathbf{b}_2 \cdots A\mathbf{b}_n]$

$\therefore A\mathbf{b}_1 = A\mathbf{b}_3$ if $\mathbf{b}_1 = \mathbf{b}_3$

(b) By the result of (a),

Columns 1 and 3 of $(AB)^T = B^T A^T$ are the same

if columns 1 and 3 of A^T are the same.

$\Rightarrow$ Rows 1 and 3 of AB are the same

if rows 1 and 3 of A are the same.

4. (7 points) Write down the 3 by 3 finite-difference matrix equation ($h = \frac{1}{4}$) for

$$-\frac{d^2u}{dx^2} + u = x, \qquad u(0) = u(1) = 0.$$

$$\frac{d^2u}{dx^2} \approx \frac{1}{h^2}\left[u(x+h) - 2u(x) + u(x-h)\right]$$ +2

$$-u(x+h) + 2u(x) - u(x-h) + h^2 u(x) = h^2 \cdot x$$

$$-u_{i-1} + (2+h^2)u_i - u_{i+1} = h^2 \cdot (ih), \quad i = 1,2,3$$ +2

$$-16u_{i-1} + 33u_i - 16u_{i+1} = \frac{i}{4}, \quad i = 1,2,3$$

$$\begin{aligned} 33u_1 - 16u_2 \phantom{{}-16u_3} &= \tfrac{1}{4} \\ -16u_1 + 33u_2 - 16u_3 &= \tfrac{2}{4} \\ -16u_2 + 33u_3 &= \tfrac{3}{4} \end{aligned}$$

+3

Equivalently,

$$\begin{bmatrix} 33 & -16 & 0 \\ -16 & 33 & -16 \\ 0 & -16 & 33 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$