Quiz #2 (CSE 400.001)

Monday, September 27, 2010

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1. (10 points) The differential equation

$$\left(x - \sqrt{x^2 + y^2}\right) dx + y\ dy = 0$$

is not exact, but show how the rearrangement

$$x\ dx + y\ dy = \sqrt{x^2 + y^2}\ dx$$

and the observation

$$\frac{1}{2}\ d(x^2 + y^2) = x\ dx + y\ dy$$

leads to a solution.

$$\frac{1}{2} d(x^2+y^2) = \sqrt{x^2+y^2}\ dx \qquad +1$$

Let $t = x^2+y^2$, then $\qquad +3$

$$\frac{1}{2} dt = \sqrt{t}\ dx \qquad +1$$

$$t^{-\frac{1}{2}} dt = 2dx \qquad +2$$

$$2\sqrt{t} = 2x + c_1 \qquad +2$$

$$\therefore \sqrt{x^2+y^2} - x = c \quad \text{for some } c = \frac{1}{2}c_1$$

+1

2. (10 points) Solve the following initial value problem:

$$y'' + 2y' + y = 4x + 2xe^{-x}, \quad y(0) = 0,\ y'(0) = 3.$$

$\lambda^2 + 2\lambda + 1 = 0$, $(\lambda+1)^2 = 0$

$y_h = c_1 e^{-x} + c_2 x e^{-x}$ (+1)

$y_p = y_{p_1} + y_{p_2}$

$y_{p_1} = K_1 x + K_0$, $y_{p_1}' = K_1$, $y_{p_1}'' = 0$

$\therefore K_1 x + 2K_1 + K_0 = 4x$, $K_1 = 4$, $K_0 = -8$

$y_{p_1} = 4x - 8$ (+2)

$y_{p_2} = \underline{Ax^3 e^{-x} + Bx^2 e^{-x}}$ (+2)

$y_{p_2}' = A(3x^2 - x^3)e^{-x} + B(2x - x^2)e^{-x}$

$y_{p_2}'' = A(6x - 6x^2 + x^3)e^{-x} + B(2 - 4x + x^2)e^{-x}$

$\therefore 6Axe^{-x} + 2Be^{-x} = 2xe^{-x}$, $A = \frac{1}{3}$, $B = 0$

$y_{p_2} = \frac{1}{3}x^3 e^{-x}$ (+2)

$y = y_h + y_p = c_1 e^{-x} + c_2 x e^{-x} + 4x - 8 + \frac{1}{3}x^3 e^{-x}$

$y(0) = c_1 - 8 = 0$ $\therefore c_1 = 8$ (+1)

$y' = -8e^{-x} + c_2 e^{-x} - c_2 x e^{-x} + 4 + x^2 e^{-x} - \frac{1}{3}x^3 e^{-x}$

$y'(0) = -8 + c_2 + 4 = 3$ $\therefore c_2 = 7$ (+1)

$\therefore y = 8e^{-x} + 7xe^{-x} + 4x - 8 + \frac{1}{3}x^3 e^{-x}$ (+1)

3. (10 points) Solve the following initial value problem:

$$x^2y'' - 3xy' + 4y = x\ln x, \quad y(1) = 1,\ y'(1) = 2.$$

$m(m-1) - 3m + 4 = 0$, $(m-2)^2 = 0$ (+1)

$y_h = c_1x^2 + c_2x^2\ln x$

$W = \begin{vmatrix} x^2 & x^2\ln x \\ 2x & 2x\ln x + x \end{vmatrix} = x^3$ (+1)

$$y_p = -x^2\int \frac{x^2\ln x \cdot \frac{x\ln x}{x^2}}{x^3}\,dx + x^2\ln x\int \frac{x^2\cdot\frac{x\ln x}{x^2}}{x^3}\,dx \quad \text{(+2)}$$

$$= -x^2\int \frac{(\ln x)^2}{x^2}\,dx + x^2\ln x\int \frac{\ln x}{x^2}\,dx$$

$$= 2x + x\ln x \quad \text{(+3)}$$

$y = y_h + y_p = c_1x^2 + c_2x^2\ln x + 2x + x\ln x$

$y(1) = c_1 + 2 = 1 \quad \therefore c_1 = -1$ (+1)

$y' = -2x + 2c_2x\ln x + c_2x + 2 + \ln x + 1$

$y'(1) = -2 + c_2 + 2 + 1 = 2 \quad \therefore c_2 = 1$ (+1)

$y = -x^2 + x^2\ln x + 2x + x\ln x$

$= -x^2 + 2x + (x + x^2)\ln x$ (+1)