Quiz #4 (CSE 400.001)

Thursday, April 29, 2004

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1. (7 points) Use $x_0 = 1.2$ and $x_1 = 1.172$ in solving the following equation by Newton's method

$$x^7 - 3 = 0.$$

How many additional iterations are necessary to produce the solution to 10D accuracy?

Solution:

$$\frac{f''(s)}{2f'(s)} \approx \frac{f''(x_1)}{2f'(x_1)} = \frac{42x_1^5}{14x_1^6} = \frac{3}{x_1} \approx 2.560$$

$$|\epsilon_{n+1}| \approx 2.560\epsilon_n^2 \approx 2.560^3\epsilon_{n-1}^4 \approx 2.560^{2^{n+1}-1}\epsilon_0^{2^{n+1}} \leq 5 \cdot 10^{-11}$$

$$\epsilon_1 - \epsilon_0 = (\epsilon_1 - s) - (\epsilon_0 - s) = -x_1 + x_0 \approx 0.028$$

$$\epsilon_1 \approx \epsilon_0 + 0.028 \approx -2.560\epsilon_0^2$$

$$2.560\epsilon_0^2 + \epsilon_0 + 0.028 \approx 0$$

$$\epsilon_0 \approx -0.03036$$

$$n = 1: \quad 2.560^3 \cdot 0.03036^4 \approx 1.425 \cdot 10^{-5} > 5 \cdot 10^{-11}$$

$$n = 2: \quad 2.560^7 \cdot 0.03036^8 \approx 5.198 \cdot 10^{-10} > 5 \cdot 10^{-11}$$

$$n = 3: \quad 2.560^{15} \cdot 0.03036^{16} < 10^{-18} < 5 \cdot 10^{-11}$$

Hence, $n = 3$ additional iterations are necessary.

2. (5 points) Interpolate

$$f_0 = f(0) = 0,\ f_1 = f(2) = 1,\ f_2 = f(4) = 6,\ f_3 = f(6) = 9$$

by the cubic spline satisfying $k_0 = 1$ and $k_3 = -5$.

Solution:

$$\begin{cases} k_0 + 4k_1 + k_2 = \frac{3}{2}\cdot(6) = 9 \\ k_1 + 4k_2 + k_3 = \frac{3}{2}\cdot(8) = 12 \end{cases} \Longrightarrow \begin{cases} 4k_1 + k_2 = 8 \\ k_1 + 4k_2 = 17 \end{cases} \Longrightarrow k_1 = 1, \quad k_2 = 4$$

$$\begin{cases} p_0(x) = \frac{1}{4}x^3 - \frac{3}{4}x^2 + x, \quad \text{for } 0 \le x \le 2 \\ p_1(x) = -\frac{3}{4}(x-2)^3 + \frac{9}{4}(x-2)^2 + x - 1, \quad \text{for } 2 \le x \le 4 \\ p_2(x) = -\frac{13}{4}(x-4)^3 + \frac{39}{4}(x-4)^2 - 5(x-4) + 6, \quad \text{for } 4 \le x \le 6 \end{cases}$$

3. (3 points) Compute the following integral using the Gauss quadrature with $n = 3$.

$$\int_2^5 \frac{2}{1+x^2}dx$$

Solution:

$$x = \frac{1}{2}(3t+7) \quad \Rightarrow \quad dx = \frac{3}{2}dt$$

$$\begin{aligned} &\int_{-1}^{1} \frac{2}{1+\frac{1}{4}(3t+7)^2} \cdot \frac{3}{2}dt \\ = &\int_{-1}^{1} \frac{12}{9t^2+42t+53}dt \\ = &\cdots \end{aligned}$$