Quiz #5 (CSE 400.001)

November 17, 2010 (Wednesday)

1. (10 points) Use $x_0 = -3$ and $x_1 = -2$ in solving the following equation by Newton's method

$$x^2 - 3 = 0.$$

How many additional iterations are necessary to produce the solution to 5D accuracy?

Solution:

$$\frac{f''(s)}{2f'(s)} \approx \frac{f''(x_1)}{2f'(x_1)} = \frac{2}{4x_1} = \frac{1}{2x_1} = -0.25$$

$$\epsilon_{n+1} \approx 0.25\epsilon_n^2 \approx 0.25^3\epsilon_{n-1}^4 \approx 0.25^{2^{n+1}-1}\epsilon_0^{2^{n+1}} \leq 5 \cdot 10^{-6}$$

$$\epsilon_1 - \epsilon_0 = (\epsilon_1 - s) - (\epsilon_0 - s) = -x_1 + x_0 = -1$$

$$\epsilon_1 = \epsilon_0 - 1 \approx 0.25\epsilon_0^2$$

$$0.25\epsilon_0^2 - \epsilon_0 + 1 \approx 0$$

$$\epsilon_0 \approx 2$$

$$\begin{array}{ll} n = 1: & 0.25^3 \cdot 2^4 \approx 2^{-2} > 5 \cdot 10^{-6} \\ n = 2: & 0.25^7 \cdot 2^8 \approx 2^{-6} > 5 \cdot 10^{-6} \\ n = 3: & 0.25^{15} \cdot 2^{16} \approx 2^{-14} > 5 \cdot 10^{-6} \\ n = 4: & 0.25^{31} \cdot 2^{32} \approx 2^{-30} < 5 \cdot 10^{-6} \end{array}$$

Hence, $n = 4$ additional iterations are necessary.

2. (5 points) Interpolate

$$f_0 = f(0) = 0,\ f_1 = f(1) = 1,\ f_2 = f(2) = 6$$

by the cubic spline satisfying $k_0 = 0$ and $k_2 = 2$.

Solution:

$$k_0 + 4k_1 + k_2 = 3 \cdot (6) = 18 \implies 4k_1 = 16 \implies k_1 = 4$$

$$\begin{cases} p_0(x) = 2x^3 - 2x^2, & \text{for } 0 \leq x \leq 1 \\ p_1(x) = -2(x-1)^3 + 3(x-1)^2 + 4(x-1) + 1, & \text{for } 2 \leq x \leq 4 \end{cases}$$

3. (5 points) Compute the following integral using the Gauss quadrature with $n = 3$.

$$\int_0^2 \frac{2}{x+1} dx$$

Solution:

$$x = t + 1 \quad \Rightarrow \quad dx = dt$$

$$\int_{-1}^1 \frac{2}{t+2} dt$$

$$= \cdots$$