Quiz #3 (CSE 400.001)

Wednesday, October 19, 2011

Name: ____________________ **E-mail:** ____________________

Dept: ____________________ **ID No:** ____________________

1. (10 points) Solve the following equation using the Power Series Method:

$$y'' - y = 0.$$

$$y = \sum_{m=0}^{\infty} a_m x^m \quad +2$$

$$y'' = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2} = \sum_{s=0}^{\infty} (s+2)(s+1) a_{s+2} x^s \quad +2$$

$$\sum_{s=0}^{\infty} \left[(s+2)(s+1) a_{s+2} - a_s \right] x^s = 0$$

$$\therefore a_{s+2} = \frac{1}{(s+2)(s+1)} a_s, \quad (s = 0, 1, 2, \cdots) \quad +2$$

$$a_2 = \frac{1}{2 \cdot 1} a_0 = \frac{1}{2!} a_0, \quad a_3 = \frac{1}{3 \cdot 2} a_1 = \frac{1}{3!} a_1$$

$$a_4 = \frac{1}{4 \cdot 3} a_2 = \frac{1}{4!} a_0, \quad a_5 = \frac{1}{5 \cdot 4} a_3 = \frac{1}{5!} a_1$$

$$\cdots \qquad \cdots$$

$$+2 \qquad +2$$

$$\therefore y = a_0 \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \right) + a_1 \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \right)$$

$$= \frac{a_0 + a_1}{2} \cdot e^{x} + \frac{a_0 - a_1}{2} \cdot e^{-x}$$

2. (10 points) Solve the following initial value problem:

$$y_1' = 5y_1 - y_2, \quad y_1(0) = 4$$
$$y_2' = 3y_1 + y_2, \quad y_2(0) = 2$$

+2

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}, \quad \det(A - \lambda I) = \lambda^2 - 6\lambda + 8 = 0$$

$$\lambda_1 = 2, \ \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}; \quad \lambda_2 = 4, \ \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

+2 +2

$$\begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x}$$

+2

$$\left.\begin{aligned} c_1 + c_2 &= 4 \\ 3c_1 + c_2 &= 2 \end{aligned}\right] \Rightarrow c_1 = -1, \ c_2 = 5$$

$$\therefore \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix} e^{2x} + \begin{bmatrix} 5 \\ 5 \end{bmatrix} e^{4x}$$

+2