Quiz #6 (CSE 400.001)

November 23, 2010 (Wednesday)

1. (6 points) It is impossible for a system of linear equations to have exactly two or exactly three solutions. Explain why by answering the following three questions:

   (a) (2 points) If (x_1, y_1, z_1) and (x_2, y_2, z_2) are two different solutions, what is another one?

   (b) (2 points) If (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3) are three different solutions, what is another one?

   (c) (2 points) If 25 planes meet at three different points, where else do they meet?

   Solution:

   (a) $\alpha(x_1, y_1, z_1) + \beta(x_2, y_2, z_2)$, where $\alpha + \beta = 1$.

   Any point on the line connecting the two points (x_1, y_1, z_1) and (x_2, y_2, z_2).

   (b) $\alpha(x_1, y_1, z_1) + \beta(x_2, y_2, z_2) + \gamma(x_3, y_3, z_3)$, where $\alpha + \beta + \gamma = 1$.

   Any point on the line/plane determined by the three points (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3).

   (c) Case 1 (Collinear points): The 25 planes meet in the line containing the three points.

   Case 2 (Non-collinear points): The 25 planes overlap in the same plane.

2. (8 points) Which rows or columns or matrices do you multiply to find

   (a) (2 points) the entry in row 3, column 4 of AB?

   (b) (2 points) the third column of AB?

   (c) (4 points) the entry in row 1, column 1 of CDE?

   Solution:

   (a) (the row 3 of A) and (the colum 4 of B)

   (b) (the matrix A) and (the third column of B)

   (c) (the row 1 of C), (the matrix D), and (the column 1 of E)

3. (6 points) Which numbers a and b lead to row exchange? Which make the matrix singular?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ a & 6 & 3 \\ 0 & b & 6 \end{bmatrix}$$

   Solution:

$$\begin{bmatrix} 1 & 2 & 3 \\ a & 6 & 3 \\ 0 & b & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 6-2a & 3-3a \\ 0 & b & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 6-2a & 3-3a \\ 0 & 0 & 6-\frac{b}{6-2a}(3-3a) \end{bmatrix}$$

   Row Exchange: $a = 3$ and $b \neq 0$

   Singular: $6 - \frac{b}{6-2a}(3-3a) = 0 \Rightarrow ab - 4a - b + 12 = 0$, which also includes the case of $a = 3$ and $b = 0$.