Quiz #7 (CSE 400.001)

November 30, 2010 (Wednesday)

Name: _____ **E-mail:** _____

Dept: _____ **ID No:** _____

1. (10 points) Diagonalize A and compute $S\Lambda^k S^{-1}$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (\lambda-2)(\lambda-1) = 0$$

$$\lambda_1 = 2,\ x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix};\quad \lambda_2 = 1,\ x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix},\ S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix},\ \Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = S\Lambda S^{-1}$$

$$A^k = S\Lambda^k S^{-1}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2^k & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2^k & 2^k-1 \\ 0 & 1 \end{bmatrix}$$