Quiz #1 (CSE 4190.313)

Monday, March 21. 2012

Name: ____________________ **E-mail:** ____________________

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1. (5 points) Suppose A is invertible and you exchange its i-th and j-th rows to reach B. Is the new matrix B invertible? Why? How would you find B^{-1} from A^{-1}?

① Yes +1

$\because$ Otherwise, $\exists\, x \neq 0$ s.t $Bx = 0$. +2

But, the same $x \neq 0$ satisfies $Ax = 0$ #

② B^{-1} can be obtained from A^{-1} by exchanging the ith and jth columns of A^{-1}. +2

2. (5 points) Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

+2

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

+1

$a \neq 0$, $a \neq b$, $b \neq c$, $c \neq d$

+2

3. (6 points) True or false (with a counterexample if false and a reason if true):

(a) (3 points) A square matrix A with a row of zeros is not invertible.

(b) (3 points) If A^T is invertible then A is invertible.

(a) True

$\ulcorner$∴ Suppose A is invertible, $\Rightarrow \exists B$ s.t $AB = I$

But, AB also has a row of zeros $\Rightarrow AB \neq I$ #

(b) True

$\ulcorner$∴ Suppose A is singular, $\Rightarrow \exists \mathbf{x} \neq \mathbf{0}$ s.t $A\mathbf{x} = \mathbf{0}$

$\Rightarrow \mathbf{x}^T A^T = \mathbf{0}^T$ and $\mathbf{x}^T = \mathbf{x}^T(A^T \cdot (A^T)^{-1}) = \mathbf{0}^T \cdot (A^T)^{-1}$

$= \mathbf{0}^T$ #

4. (4 points) True or false? Give a specific counterexample when false.

(a) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.

(b) If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.

(c) If rows 1 and 3 of A are the same, so are rows 1 and 3 of AB.

(d) $(AB)^2 = A^2B^2$.

(a), (c) True

(b) False.

$\ulcorner$Counter-example

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad AB = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

(d) False

$\ulcorner$Counter-example $A = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow A^2B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}, \quad (AB)^2 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \therefore (AB)^2 \neq A^2B^2$$