Quiz #2 (CSE 4190.313)

Monday, March 28, 2011

Name: _____ **E-mail:** _____

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1. (5 points) Find L and U for the nonsymmetric matrix

$$A = \begin{bmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{bmatrix}$$

$$\begin{bmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{bmatrix} \rightarrow \begin{bmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & b-r & c-r & t-r \\ 0 & b-r & c-r & d-r \end{bmatrix} \rightarrow \begin{bmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & c-s & d-s \end{bmatrix}$$

$l_{21} = l_{31} = l_{41} = 1$

$l_{32} = l_{42} = 1$

$$\rightarrow \begin{bmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & 0 & d-t \end{bmatrix}$$

$l_{43} = 1$

U (+2)

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(+3)

2. (5 points) Write down the 3 by 3 finite-difference matrix equation ($h = \frac{1}{4}$) for

$$-\frac{d^2u}{dx^2} + u = x, \quad u(0) = u(1) = 0.$$

$$\frac{-u_{i-1} + 2u_i - u_{i+1}}{h^2} + u_i = ih \quad (+1)$$

$$-16u_{i-1} + 33u_i - 16u_{i+1} = \frac{i}{4} \quad (+1)$$

$$\begin{cases} 33u_1 - 16u_2 = \frac{1}{4} \\ -16u_1 + 33u_2 - 16u_3 = \frac{2}{4} \\ -16u_2 + 33u_3 = \frac{3}{4} \end{cases} \quad (+3)$$

3. (5 points) Find the inverse of A

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 & 0 \\ -\frac{1}{3} & 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{3} & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

(+1) (+1) (+1)

$$A^{-1} = GFE = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & -\frac{1}{2} & 1 \end{bmatrix} \quad (+2)$$