Chapter 3
Graphics Output Primitives

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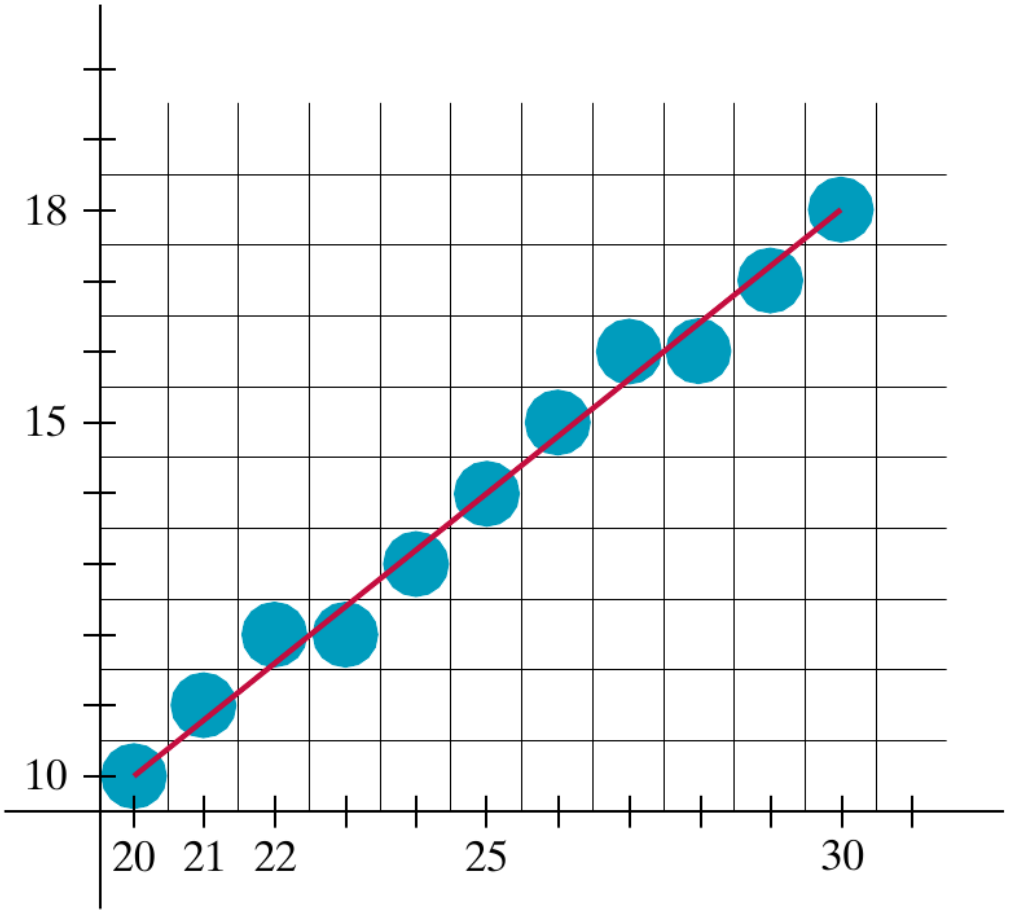
Line Drawing



Line Drawing Algorithms

$$y = m \cdot x + b \qquad (x_0, y_0) \text{ and } (x_{\text{end}}, y_{\text{end}})$$

$$m = \frac{y_{\text{end}} - y_0}{x_{\text{end}} - x_0} \qquad b = y_0 - m \cdot x_0$$

$$\delta y = m \cdot \delta x \qquad y_{k+1} = y_k + m$$

$$\delta x = \frac{\delta y}{m} \qquad x_{k+1} = x_k + \frac{1}{m}$$

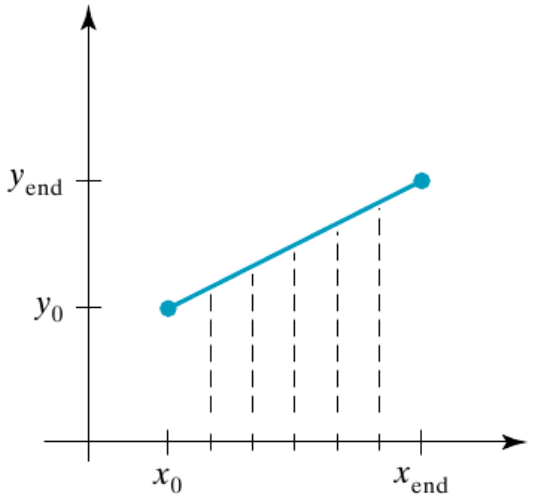


FIGURE 3–7 Straight-line segment with five sampling positions along the x axis between x_0 and x_{end}.

DDA Algorithm

```
void lineDDA (int x0, int y0, int xEnd, int yEnd)
{
    int dx = xEnd - x0,  dy = yEnd - y0,  steps,  k;
    float xIncrement, yIncrement, x = x0, y = y0;

    if (fabs (dx) > fabs (dy))
       steps = fabs (dx);
    else
       steps = fabs (dy);
    xIncrement = float (dx) / float (steps);
    yIncrement = float (dy) / float (steps);

    setPixel (round (x), round (y));
    for (k = 0; k < steps; k++) {
       x += xIncrement;
       y += yIncrement;
       setPixel (round (x), round (y));
    }
}
```

Bresenham Algorithm

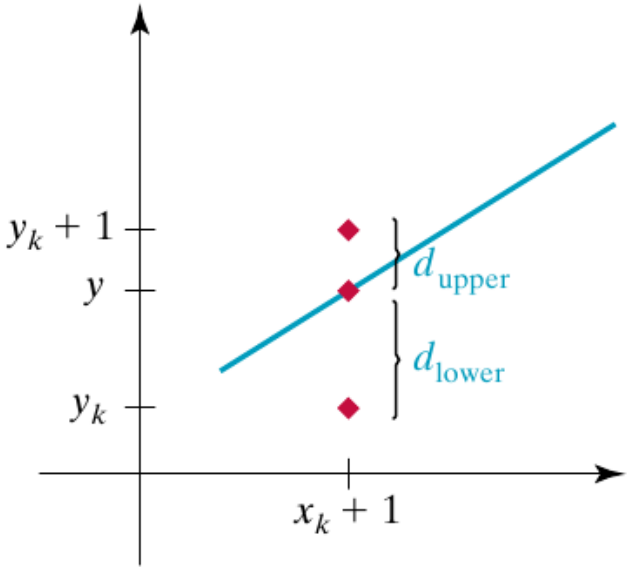
$$y = m(x_k + 1) + b$$

$$d_{\text{lower}} = y - y_k$$

$$= m(x_k + 1) + b - y_k$$

$$d_{\text{upper}} = (y_k + 1) - y$$

$$= y_k + 1 - m(x_k + 1) - b$$



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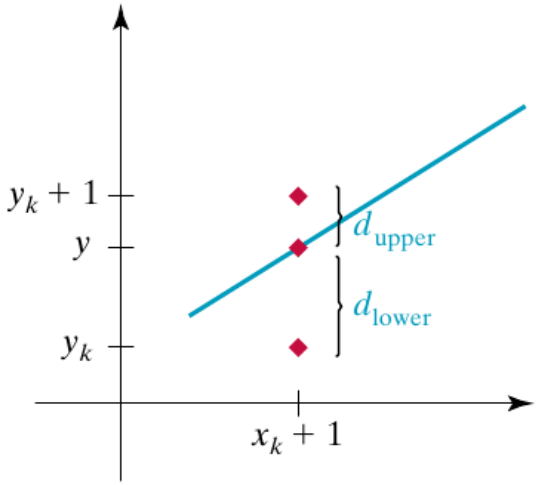


$$d_{\text{lower}} - d_{\text{upper}} = 2m(x_k + 1) - 2y_k + 2b - 1$$

$$p_k = \Delta x(d_{\text{lower}} - d_{\text{upper}})$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

Bresenham Algorithm

$$d_{\text{lower}} - d_{\text{upper}} = 2m(x_k + 1) - 2y_k + 2b - 1$$

$$p_k = \Delta x(d_{\text{lower}} - d_{\text{upper}})$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

$$p_0 = 2\Delta y - \Delta x$$

Bresenham's Line-Drawing Algorithm for $|m| < 1.0$

1. Input the two line endpoints and store the left endpoint in (x_0, y_0).
2. Set the color for frame-buffer position (x_0, y_0); i.e., plot the first point.
3. Calculate the constants Δx, Δy, $2\Delta y$, and $2\Delta y - 2\Delta x$, and obtain the starting value for the decision parameter as

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k = 0$, perform the following test. If $p_k < 0$, the next point to plot is $(x_k + 1, y_k)$ and

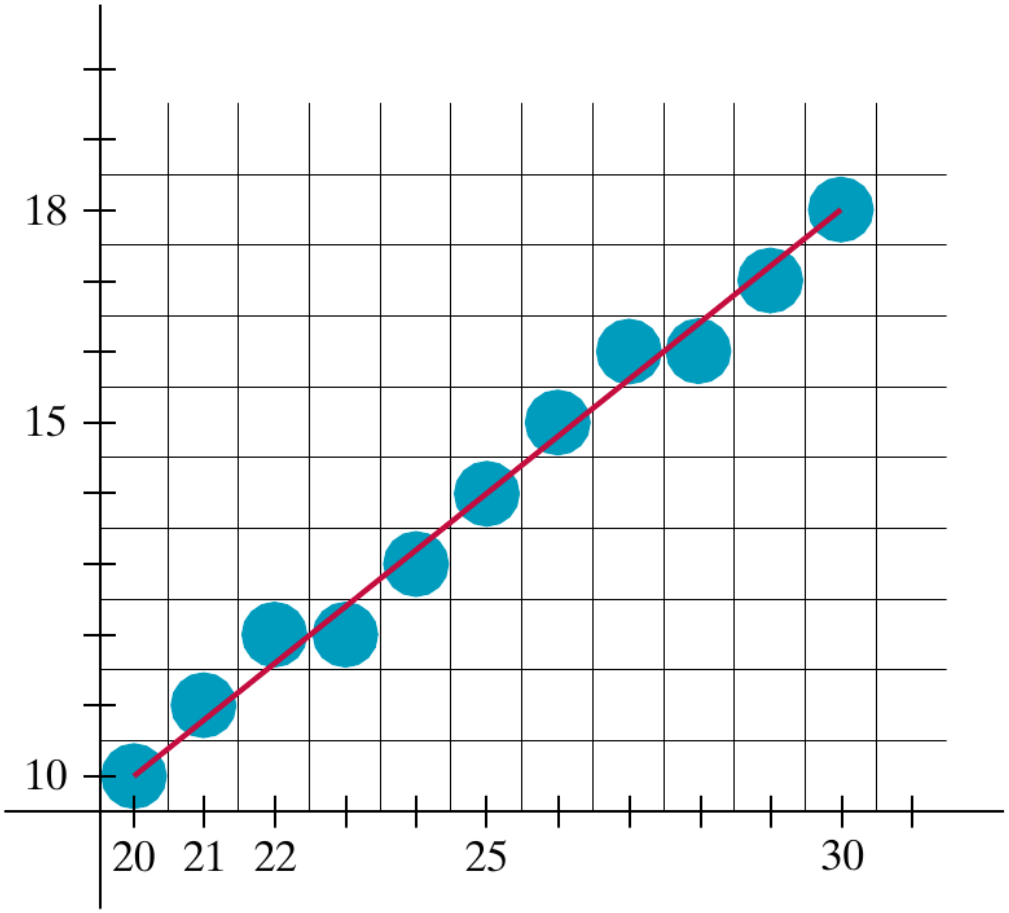
$$p_{k+1} = p_k + 2\Delta y$$

   Otherwise, the next point to plot is $(x_k + 1, y_k + 1)$ and

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Perform step 4 $\Delta x - 1$ times.

Bresenham Algorithm



Pixel Addressing

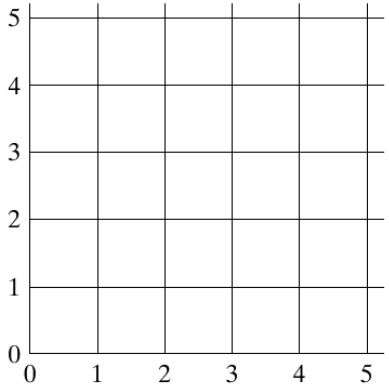


FIGURE 3-33 Lower-left section of a screen area with coordinate positions referenced by grid intersection lines.

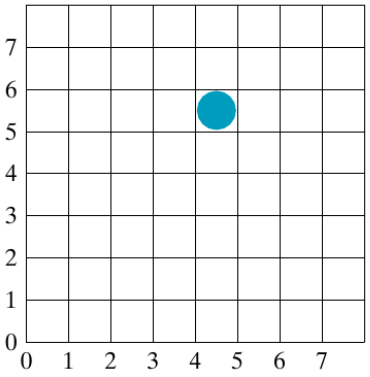


FIGURE 3-34 Illuminated pixel at raster position (4, 5).

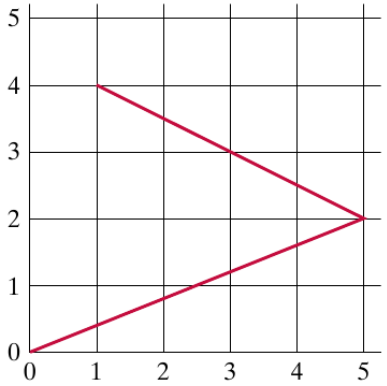


FIGURE 3-35 Line path for two connected line segments between screen grid-coordinate positions.

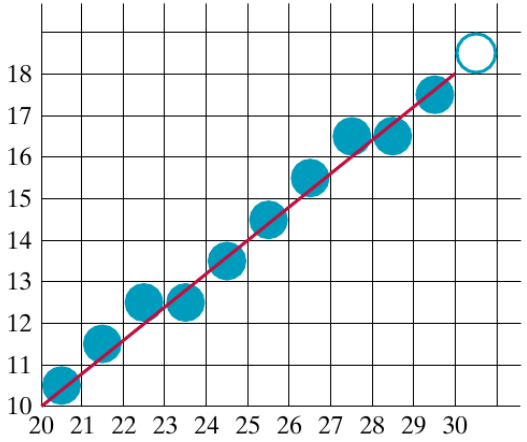
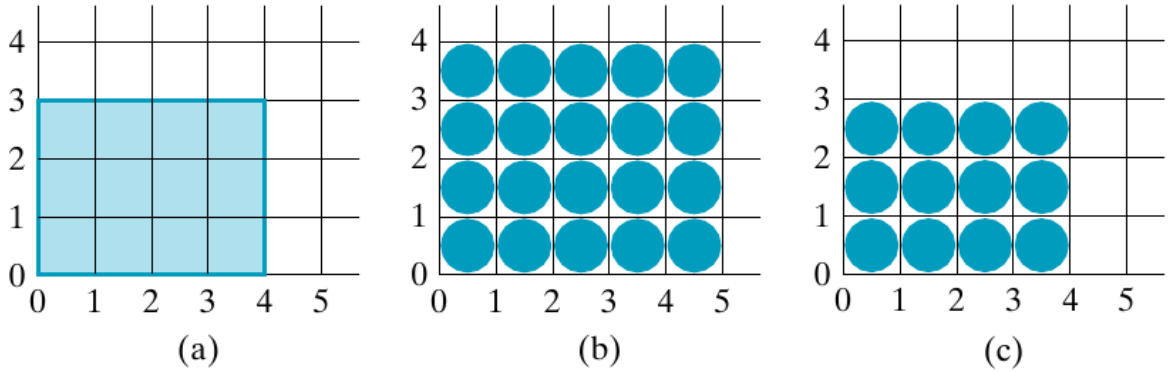


FIGURE 3-36 Line path and corresponding pixel display for grid endpoint coordinates (20, 10) and (30, 18).

FIGURE 3-37
Conversion of rectangle (a) with vertices at screen coordinates (0, 0), (4, 0), (4, 3), and (0, 3) into display (b) that includes the right and top boundaries and into display (c) that maintains geometric magnitudes.



Pixel Addressing

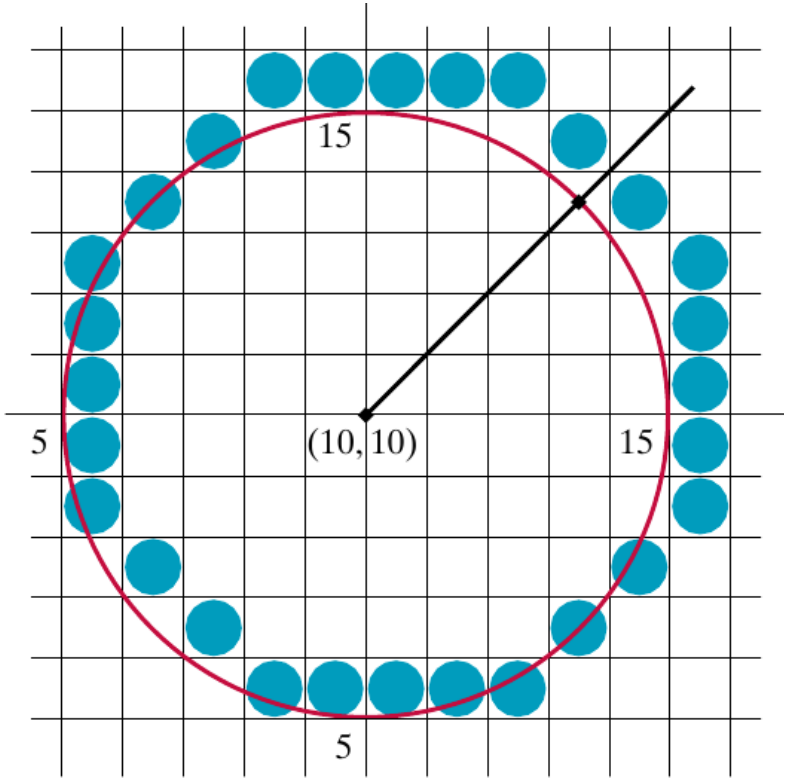


FIGURE 3-38
A midpoint-algorithm plot of the circle equation $(x - 10)^2 + (y - 10)^2 = 5^2$ using pixel-center coordinates.

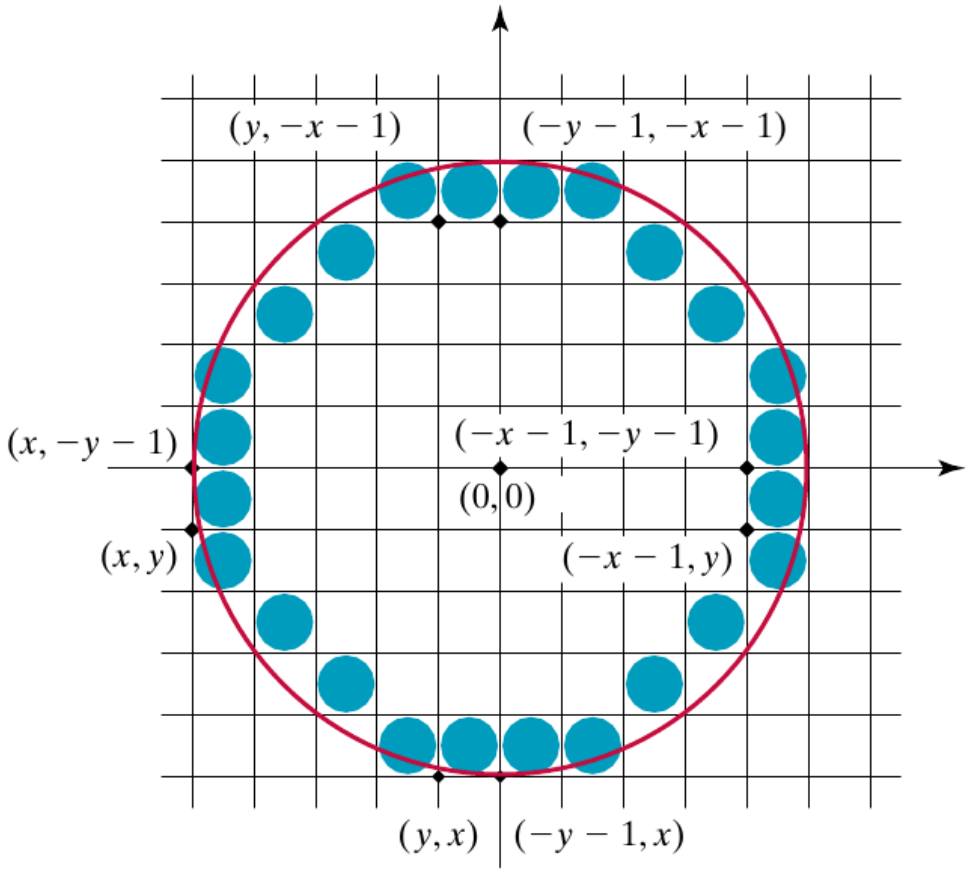


FIGURE 3-39
Modification of the circle plot in Fig. 3-38 to maintain the specified circle diameter of 10.

Fill-Area Primitives

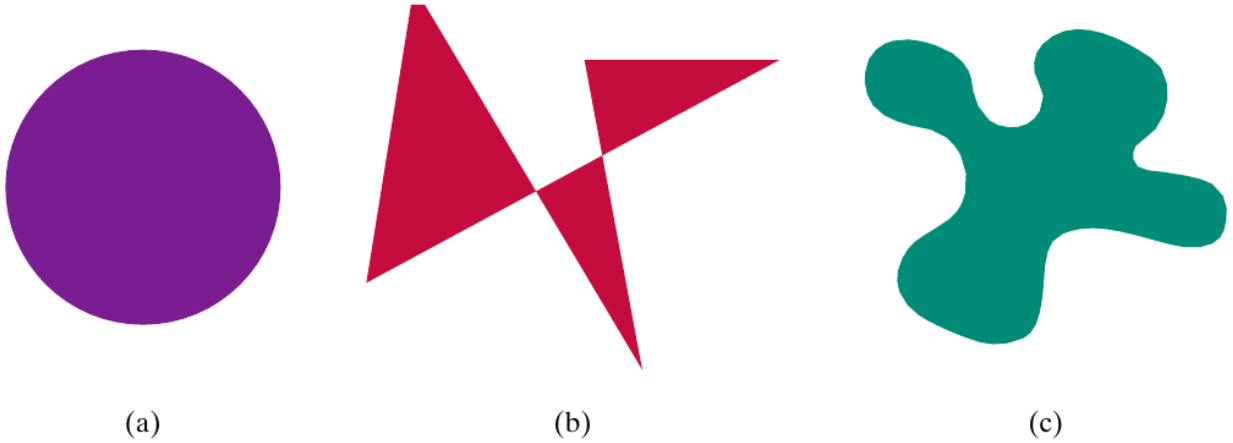


FIGURE 3-40 Solid-color fill areas specified with various boundaries. (a) A circular fill region. (b) A fill area bounded by a closed polyline. (c) A filled area specified with an irregular curved boundary.

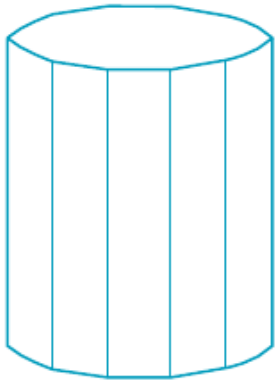


FIGURE 3-41 Wire-frame representation for a cylinder, showing only the front (visible) faces of the polygon mesh used to approximate the surfaces.

Polygon Fill-Areas

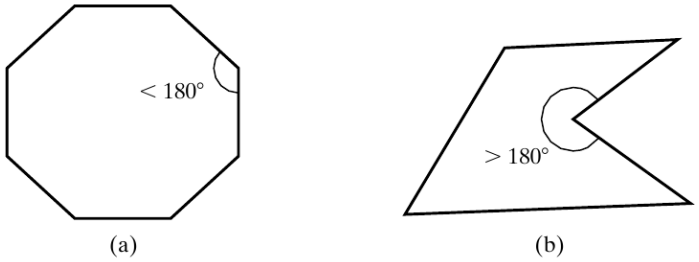
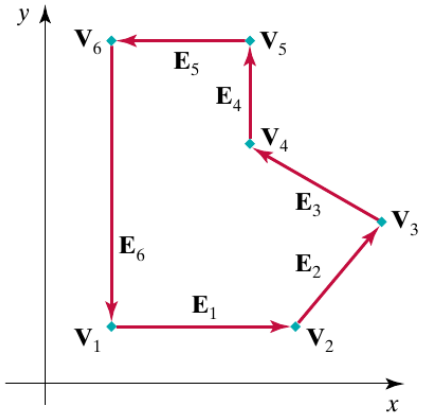


FIGURE 3-42 A convex polygon (a), and a concave polygon (b).



$(\mathbf{E}_1 \times \mathbf{E}_2)_z > 0$

$(\mathbf{E}_2 \times \mathbf{E}_3)_z > 0$

$(\mathbf{E}_3 \times \mathbf{E}_4)_z < 0$

$(\mathbf{E}_4 \times \mathbf{E}_5)_z > 0$

$(\mathbf{E}_5 \times \mathbf{E}_6)_z > 0$

$(\mathbf{E}_6 \times \mathbf{E}_1)_z > 0$

FIGURE 3-43 Identifying a concave polygon by calculating cross products of successive pairs of edge vectors.

삼각형이나 convex polygon들로 분할하는 최적화된 알고리즘들은 Computational Geometry 분야에서 아주 많이 연구되었다. 여기서는 non-convex한 vertex를 간단하게 찾는 알고리즘을 소개한다.

Splitting Concave Polygons

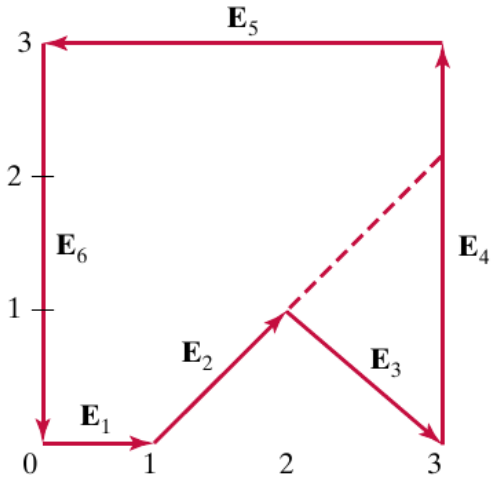


FIGURE 3-44 Splitting a concave polygon using the vector method.

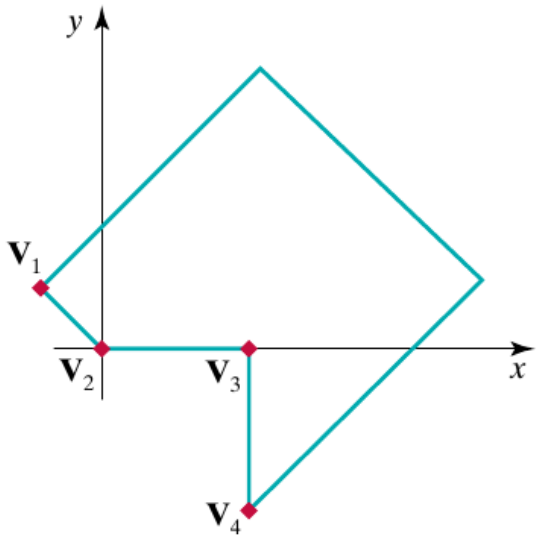
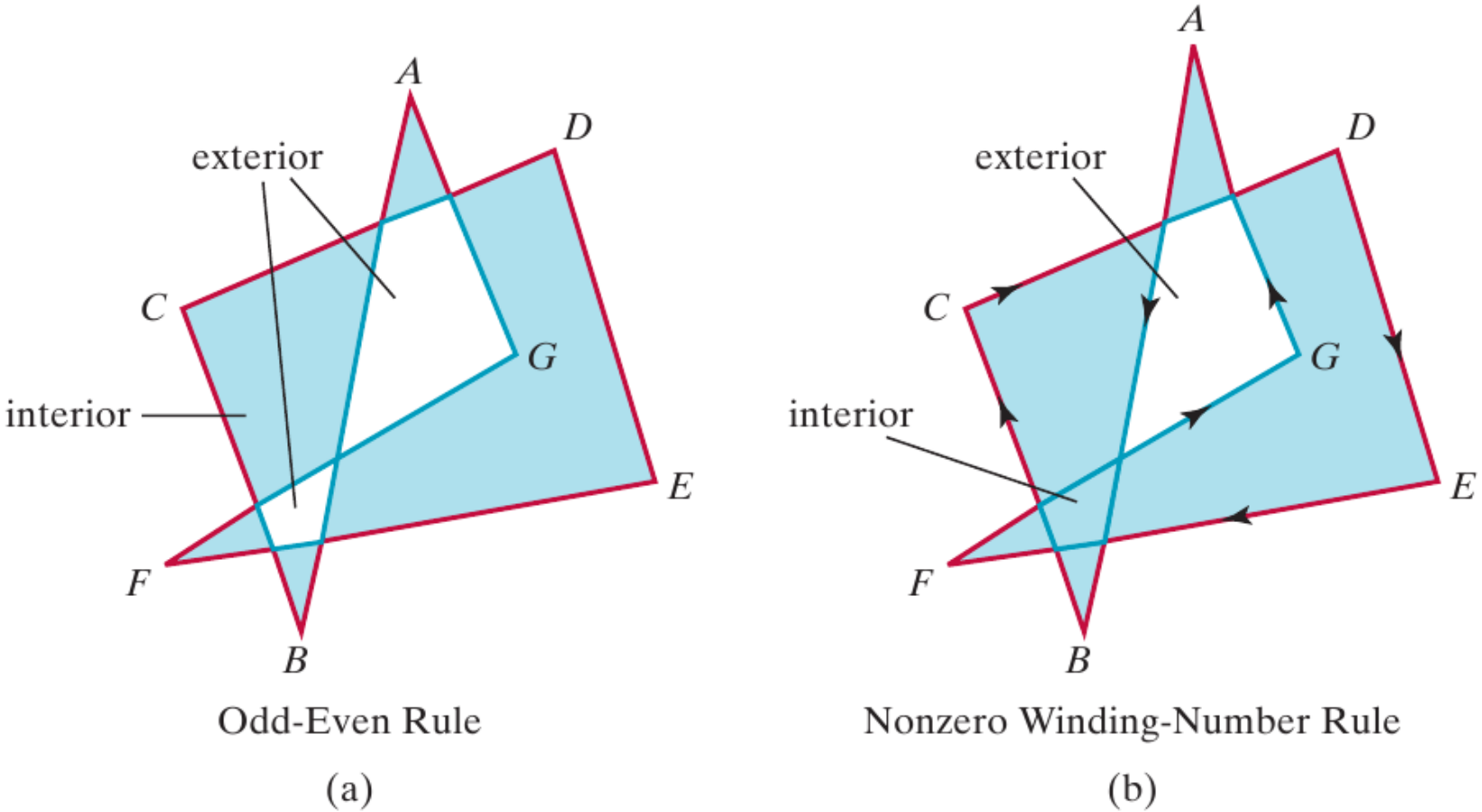


FIGURE 3-45 Splitting a concave polygon using the rotational method. After moving $\mathbf{V}_2$ to the coordinate origin and rotating $\mathbf{V}_3$ onto the x axis, we find that $\mathbf{V}_4$ is below the x axis. So we split the polygon along the line of $\overline{\mathbf{V}_2\mathbf{V}_3}$, which is the x axis.

오른쪽의 알고리즘은 전체의 다각형을 변환하는 과정이 비효율적이다.
E2 를 지나는 직선을 구하고 다음에 연속적으로 나오는 vertex들을 직선의 식에 넣어보면서 직선방정식의 부호가 바뀌는 부분에서 두 직선의 교점을 구하면 E4가 분할되는 교점의 좌표를 쉽게 구할 수 있다.

Inside-Outside Tests



그림(b)의 진행방향을 반대로 하여 시계반대방향으로 진행하는 것이 주로 사용된다. 다음 페이지의 예에서는 시계반대방향으로 진행한다.

Boolean Ops using Winding #s

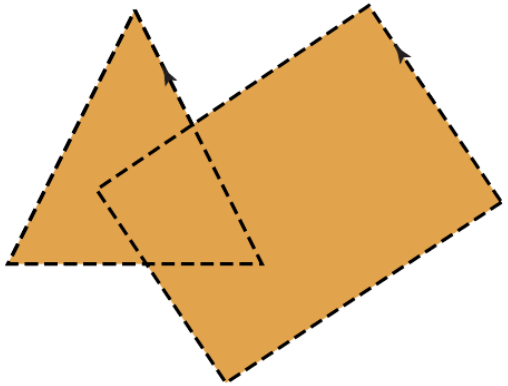


FIGURE 3-47 A fill area defined as a region that has a positive value for the winding number. This fill area is the union of two regions, each with a counterclockwise border direction.

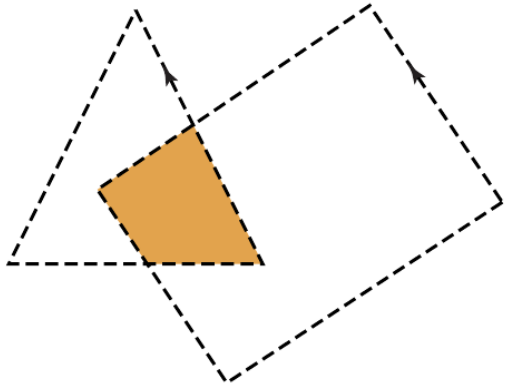


FIGURE 3-48 A fill area defined as a region with a winding number greater than 1. This fill area is the intersection of two regions, each with a counterclockwise border direction.

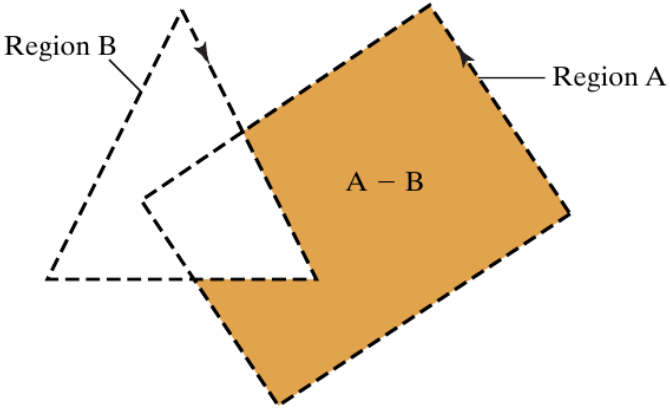
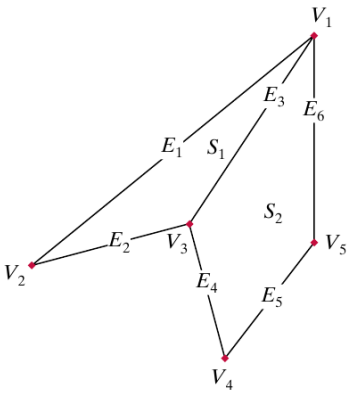


FIGURE 3-49 A fill area defined as a region with a positive value for the winding number. This fill area is the difference, A − B, of two regions, where region A has a positive border direction (counterclockwise) and region B has a negative border direction (clockwise).

Polygon Tables

E_1:	V_1, V_2, S_1
E_2:	V_2, V_3, S_1
E_3:	V_3, V_1, S_1, S_2
E_4:	V_3, V_4, S_2
E_5:	V_4, V_5, S_2
E_6:	V_5, V_1, S_2

FIGURE 3–51 Edge table expanded to include pointers into the surface-facet table.



Listing the geometric data in three tables, as in Fig. 3-50, provides a convenient reference to the individual components (vertices, edges, and surface facets) for each object. Also, the object can be displayed efficiently by using data from the edge table to identify polygon boundaries. An alternative arrangement is to use just two tables: a vertex table and a surface-facet table. But this scheme is less convenient, and some edges could get drawn twice in a wire-frame display. Another possibility is to use only a surface-facet table, but this duplicates coordinate information, since explicit coordinate values are listed for each vertex in each polygon facet. Also the relationship between edges and facets would have to be reconstructed from the vertex listings in the surface-facet table.

We can add extra information to the data tables of Fig. 3-50 for faster information extraction. For instance, we could expand the edge table to include forward pointers into the surface-facet table so that a common edge between polygons could be identified more rapidly (Fig. 3-51). This is particularly useful for rendering procedures that must vary surface shading smoothly across the edges from one polygon to the next. Similarly, the vertex table could be expanded to reference corresponding edges, for faster information retrieval.

VERTEX TABLE	
V_1:	x_1, y_1, z_1
V_2:	x_2, y_2, z_2
V_3:	x_3, y_3, z_3
V_4:	x_4, y_4, z_4
V_5:	x_5, y_5, z_5

EDGE TABLE	
E_1:	V_1, V_2
E_2:	V_2, V_3
E_3:	V_3, V_1
E_4:	V_3, V_4
E_5:	V_4, V_5
E_6:	V_5, V_1

SURFACE-FACET TABLE	
S_1:	E_1, E_2, E_3
S_2:	E_3, E_4, E_5, E_6

삼각형의 개수가 많아지면, vertex와 edge의 개수도 증가하고, 따라서 vertex와 edge들의 index를 표시하는 정보의 양이 크게 증가한다. 이는 심각한 문제이다.

Plane Equations

$$A\,x + B\,y + C\,z + D = 0 \tag{3-59}$$

$$(A/D)x_k + (B/D)y_k + (C/D)z_k = -1, \qquad k = 1, 2, 3 \tag{3-60}$$

Cramer's rule, as

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \qquad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix}$$

$$C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \tag{3-61}$$

Expanding the determinants, we can write the calculations for the plane coefficients in the form

$$\begin{aligned} A &= y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2) \\ B &= z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2) \\ C &= x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \\ D &= -x_1(y_2z_3 - y_3z_2) - x_2(y_3z_1 - y_1z_3) - x_3(y_1z_2 - y_2z_1) \end{aligned} \tag{3-62}$$

Front and Back Polygon Faces

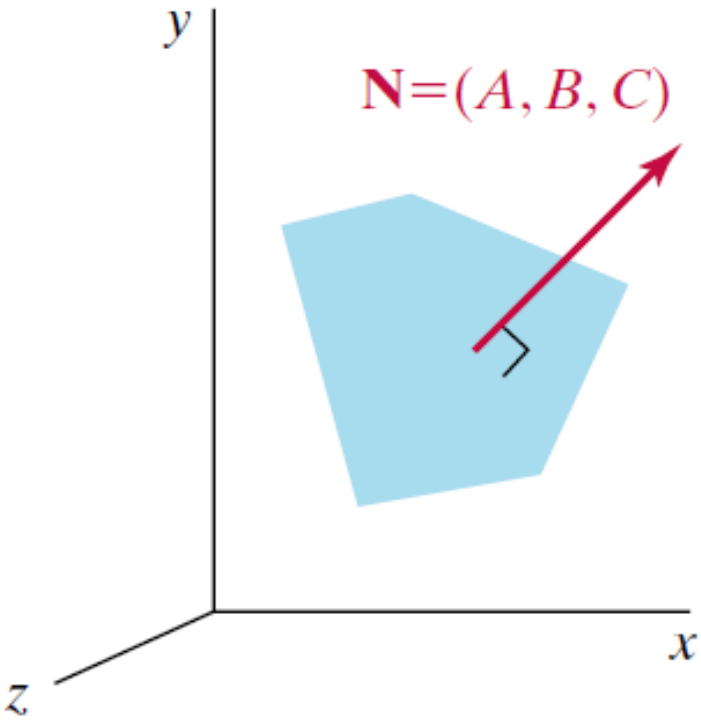


FIGURE 3-53 The normal vector **N** for a plane described with the equation $Ax + By + Cz + D = 0$ is perpendicular to the plane and has Cartesian components (A, B, C).

if $A\,x + B\,y + C\,z + D < 0$, behind the plane

if $A\,x + B\,y + C\,z + D > 0$, in front of the plane