Chapter 7
Three-Dimensional Viewing

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3D Viewing Pipeline

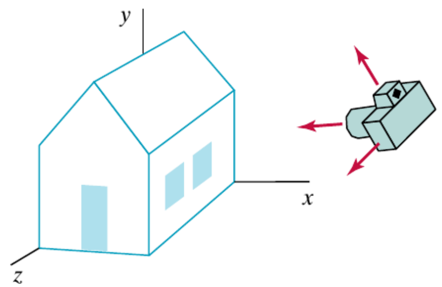


FIGURE 7-10 Photographing a scene involves selection of the camera position and orientation.

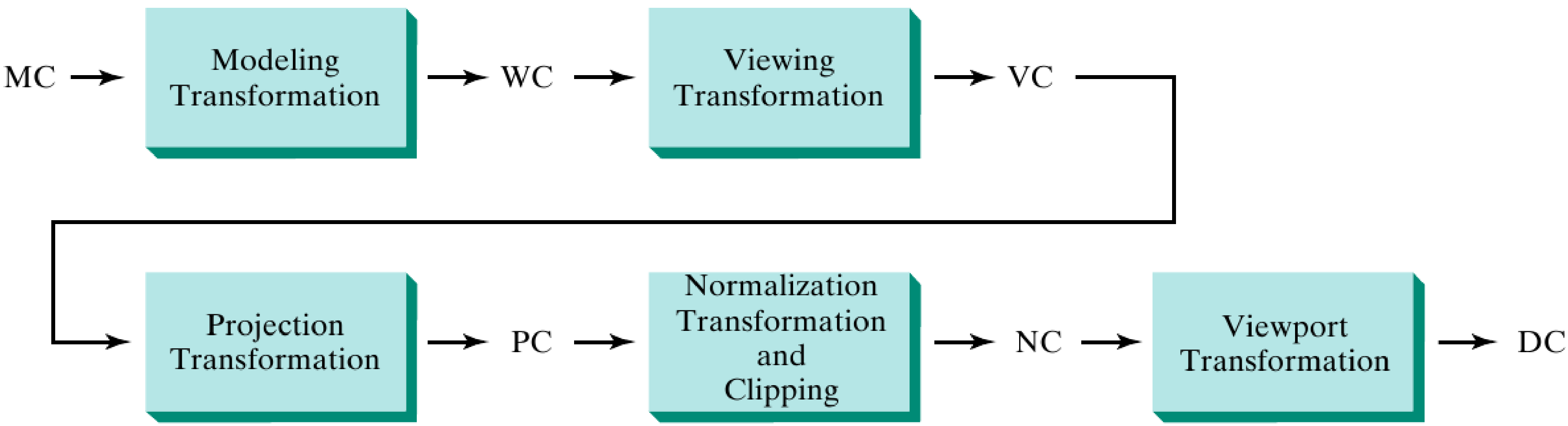


FIGURE 7-11 General three-dimensional transformation pipeline, from modeling coordinates to world coordinates to viewing coordinates to projection coordinates to normalized coordinates and, ultimately, to device coordinates.

3D Viewing Parameters

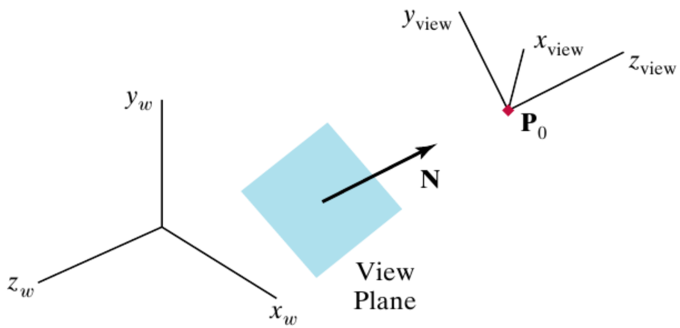


FIGURE 7–13 Orientation of the view plane and view-plane normal vector **N**.

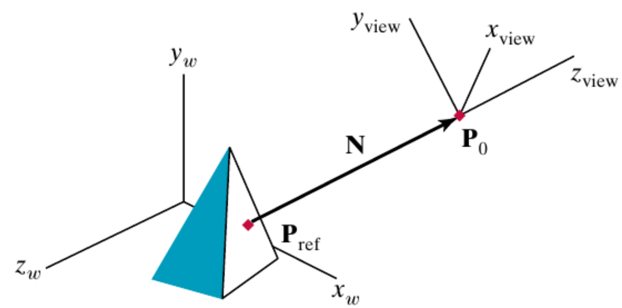


FIGURE 7–15 Specifying the view-plane normal vector **N** as the direction from a selected reference point $\mathbf{P}_{ref}$ to the viewing-coordinate origin $\mathbf{P}_0$.

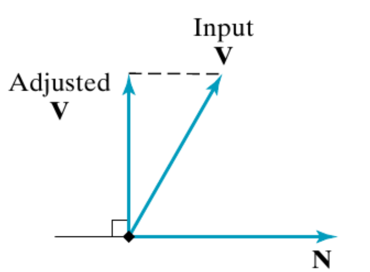


FIGURE 7–16 Adjusting the input direction of the view-up vector **V** to an orientation perpendicular to the view-plane normal vector **N**.

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = (n_x,\ n_y,\ n_z)$$

$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{n}}{|\mathbf{V}|} = (u_x,\ u_y,\ u_z)$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u} = (v_x,\ v_y,\ v_z)$$

$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{n}}{|\mathbf{V} \times \mathbf{n}|} = (u_x,\ u_y,\ u_z)$$

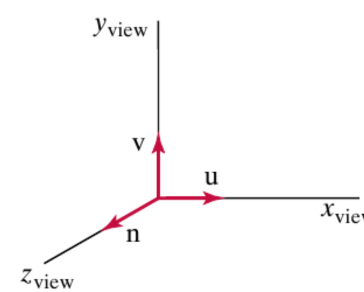


FIGURE 7–17 A right-handed viewing system defined with unit vectors **u**, **v**, and **n**.

3D Projections

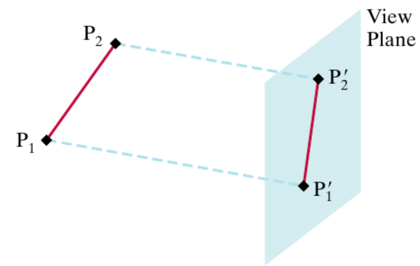


FIGURE 7-22 Parallel projection of a line segment onto a view plane.

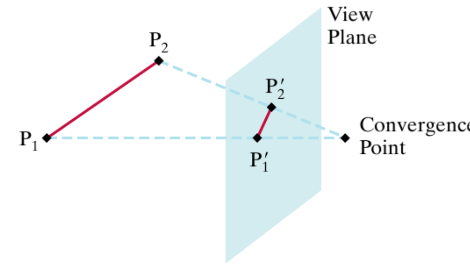


FIGURE 7-23 Perspective projection of a line segment onto a view plane.

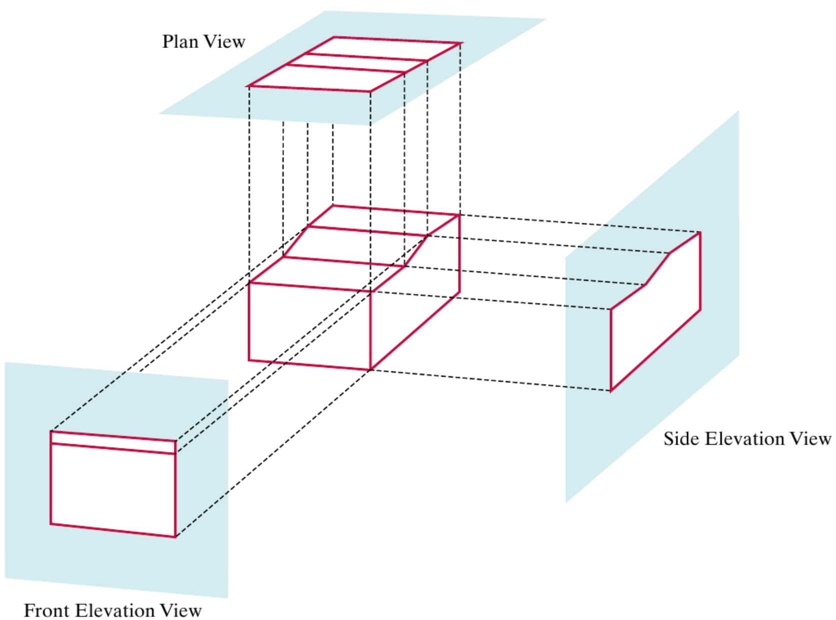


FIGURE 7-24 Orthogonal projections of an object, displaying plan and elevation views.

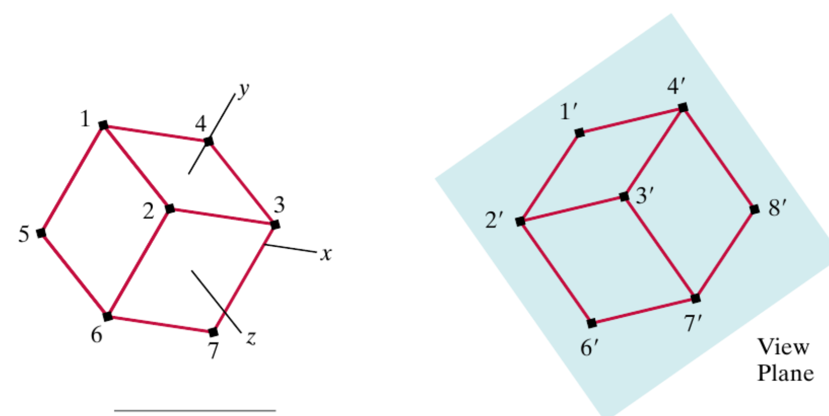


FIGURE 7-25 An isometric projection of a cube.

Orthogonal Projection

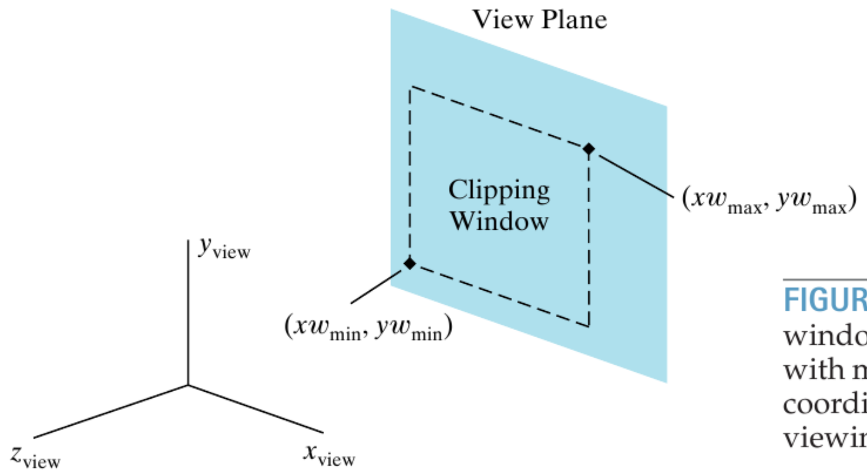


FIGURE 7-27 A clipping window on the view plane, with minimum and maximum coordinates given in the viewing reference system.

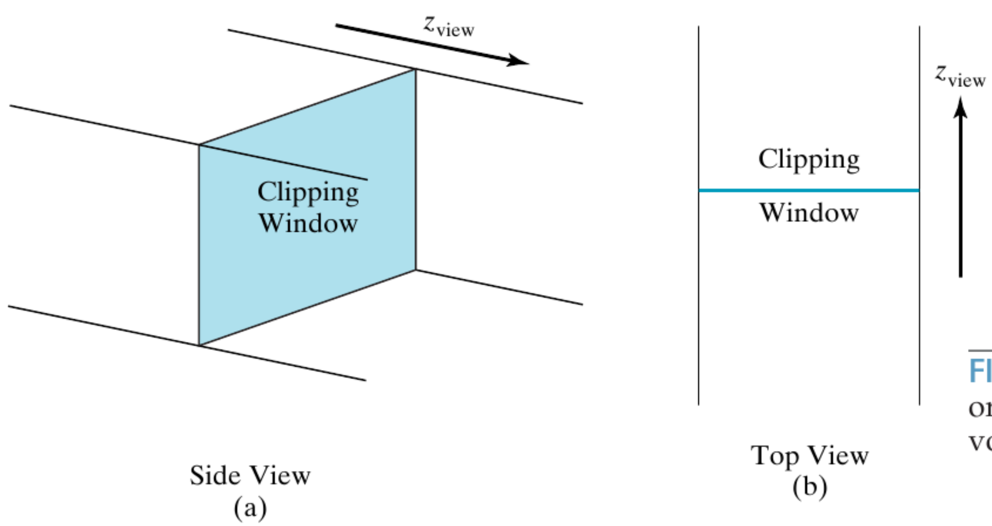


FIGURE 7-28 Infinite orthogonal-projection view volume.

Orthogonal Projection

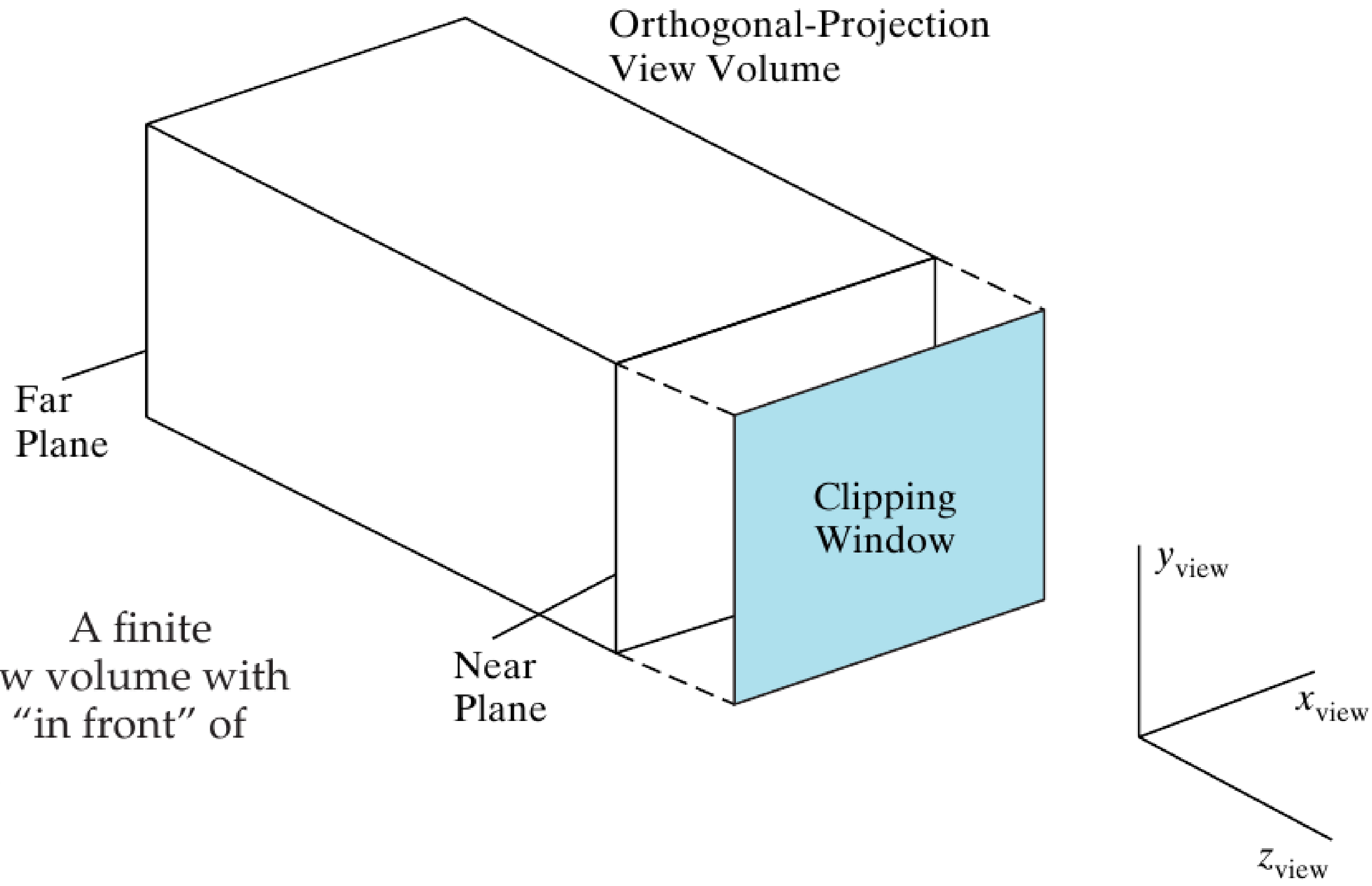


FIGURE 7-29 A finite orthogonal view volume with the view plane "in front" of the near plane.

Orthogonal Projection

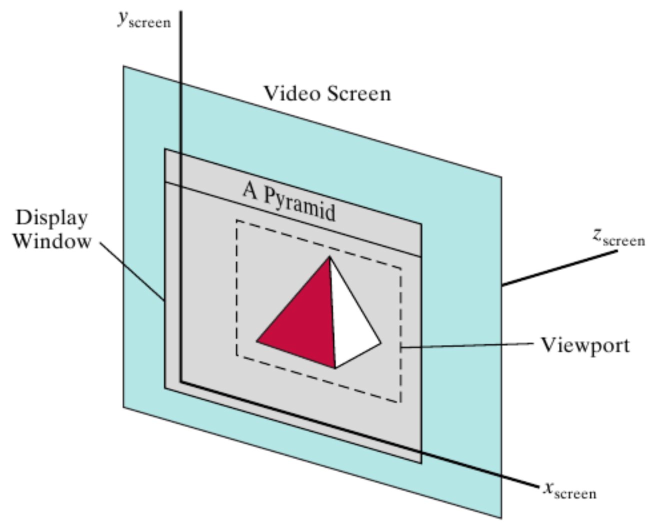


FIGURE 7–30 A left-handed screen-coordinate reference frame.

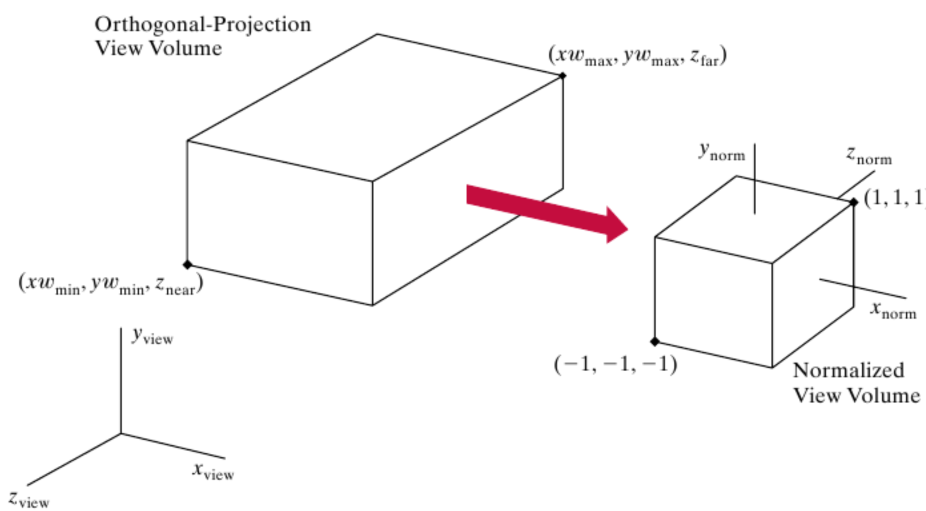


FIGURE 7–31 Normalization transformation from an orthogonal-projection view volume to the symmetric normalization cube within a left-handed reference frame.

Oblique Parallel Projection

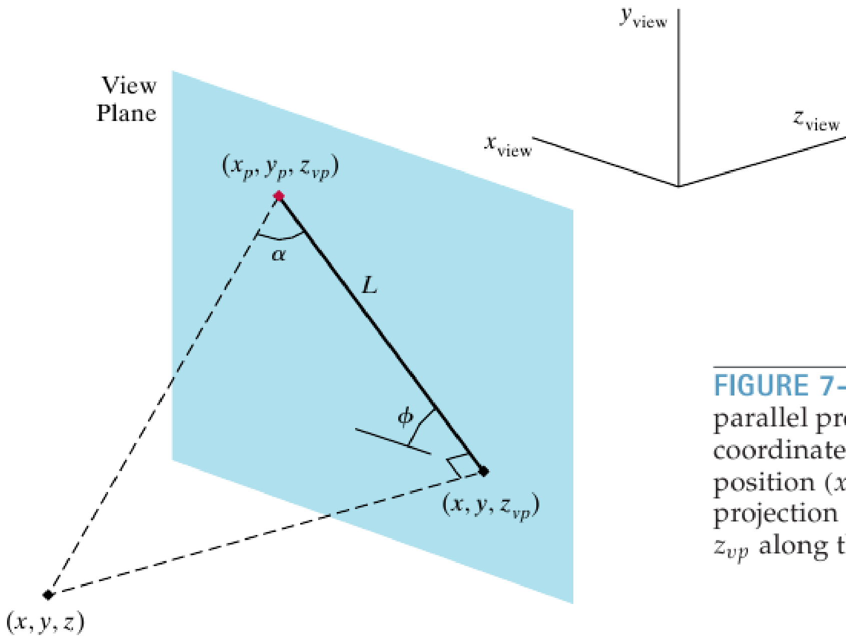
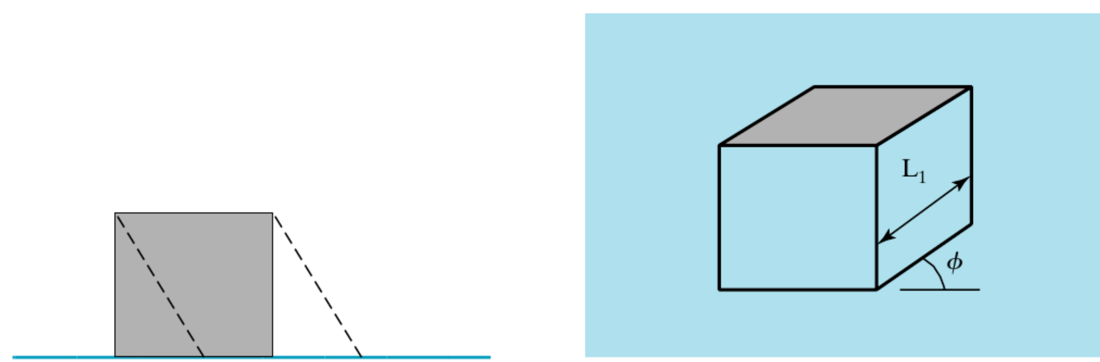


FIGURE 7-33 Oblique parallel projection of coordinate position (x, y, z) to position (x_p, y_p, z_{vp}) on a projection plane at position z_{vp} along the z_{view} axis.

$$x_p = x + L\cos\phi$$

$$y_p = y + L\sin\phi$$



View Plane (a) View Plane (b)

Oblique Parallel Projection

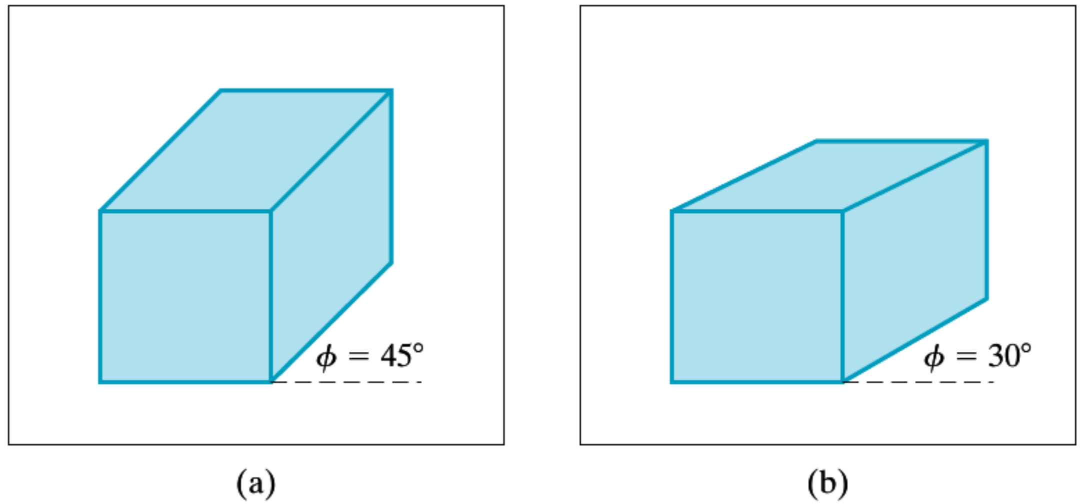


FIGURE 7-35 Cavalier projections of a cube onto a view plane for two values of angle ϕ. The depth of the cube is projected with a length equal to that of the width and height.

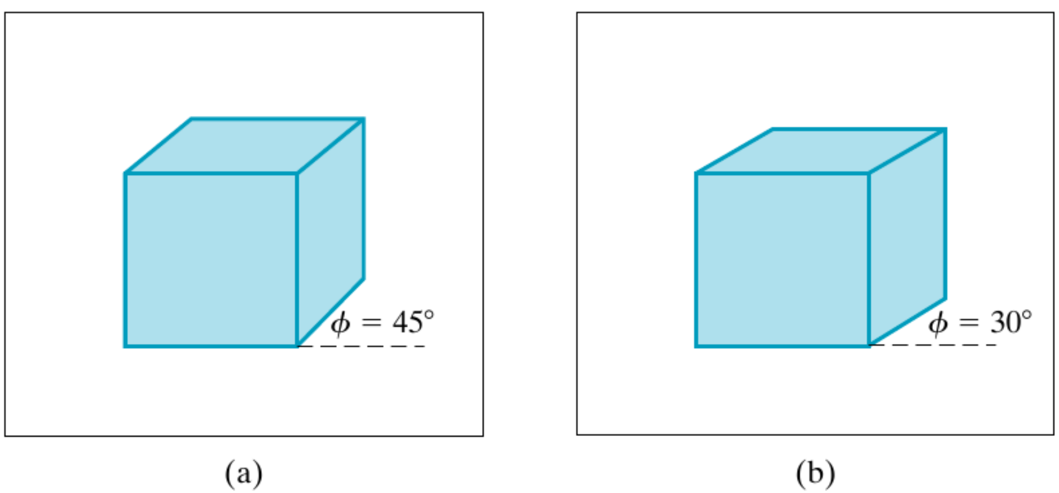


FIGURE 7-36 Cabinet projections of a cube onto a view plane for two values of angle ϕ. The depth is projected with a length that is one half that of the width and height of the cube.

Oblique Parallel Projection

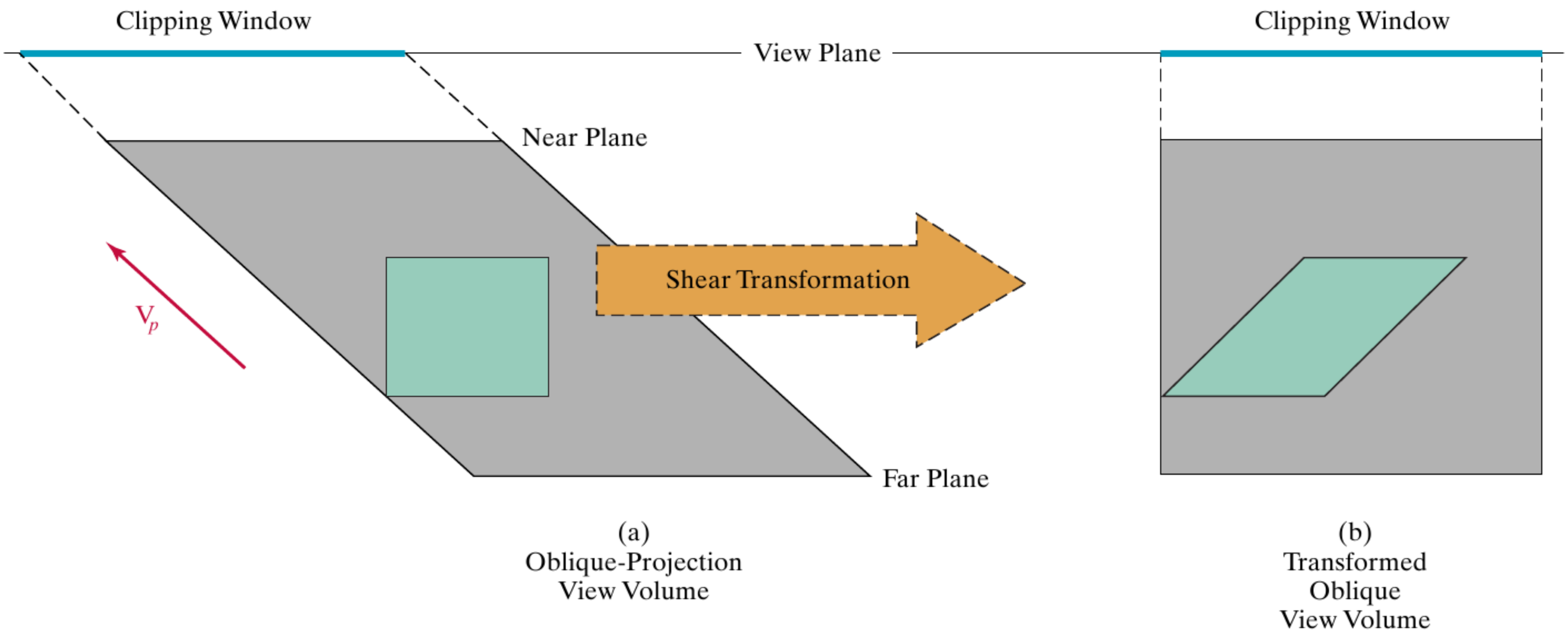


FIGURE 7–39 Top view of an oblique parallel-projection transformation. The oblique view volume is converted into a rectangular parallelepiped, and objects in the view volume, such as the green block, are mapped to orthogonal-projection coordinates.

3D Perspective Projection

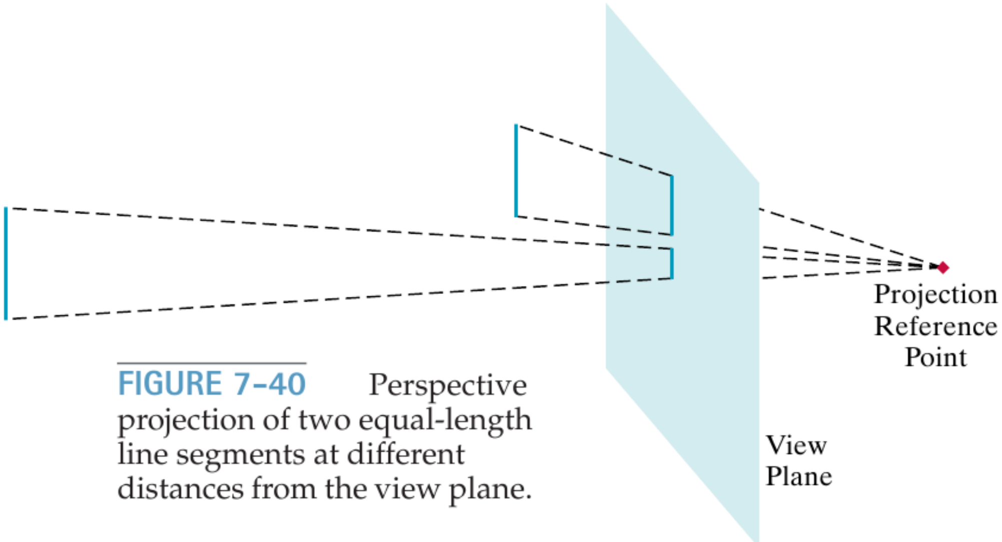


FIGURE 7-40 Perspective projection of two equal-length line segments at different distances from the view plane.

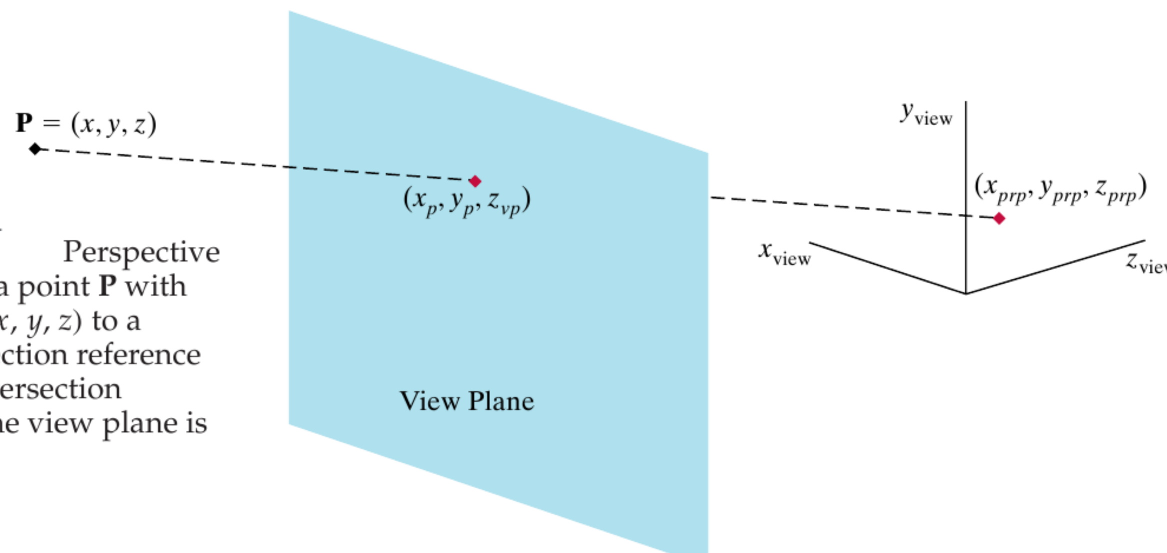


FIGURE 7-41 Perspective projection of a point **P** with coordinates (x, y, z) to a selected projection reference point. The intersection position on the view plane is (x_p, y_p, z_{vp}).

3D Perspective Projection

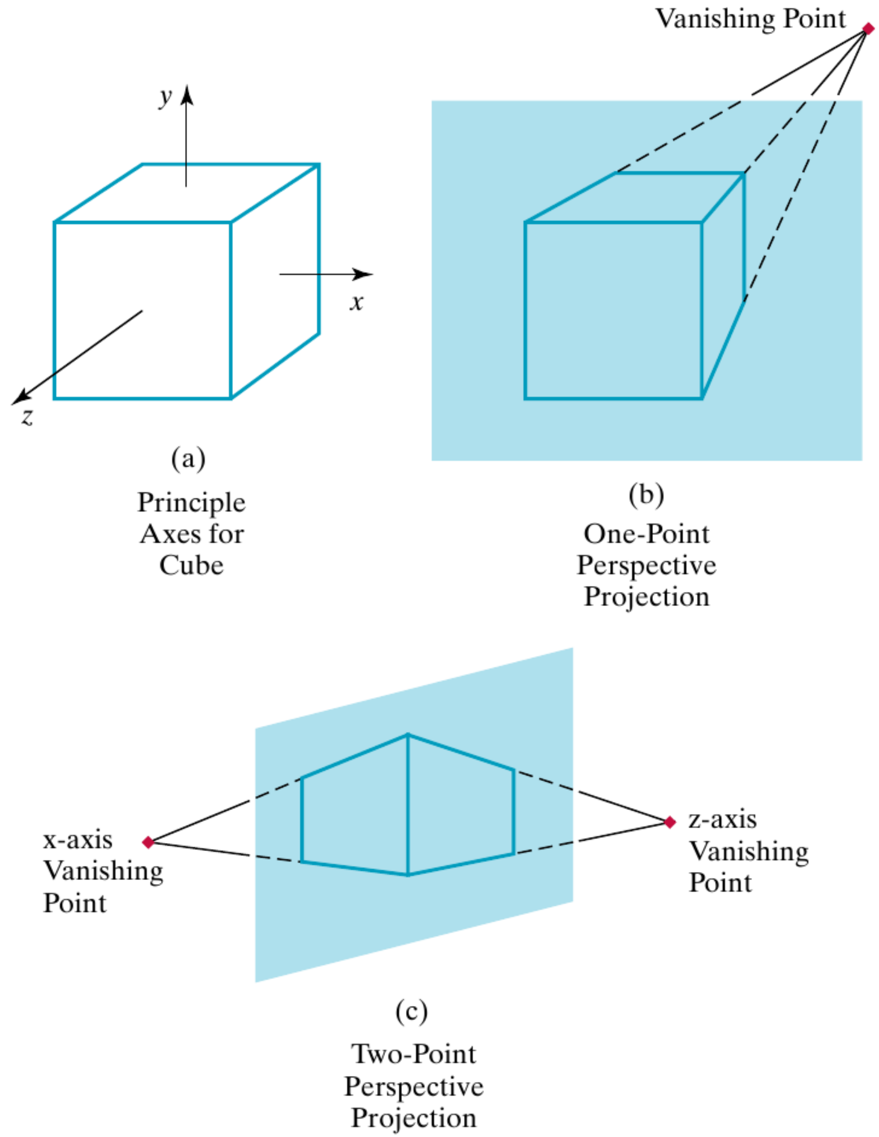


FIGURE 7-44 Principal vanishing points for perspective-projection views of a cube. When the cube in (a) is projected to a view plane that intersects only the *z* axis, a single vanishing point in the *z* direction (b) is generated. When the cube is projected to a view plane that intersects both the *z* and *x* axes, two vanishing points (c) are produced.

3D Perspective Projection

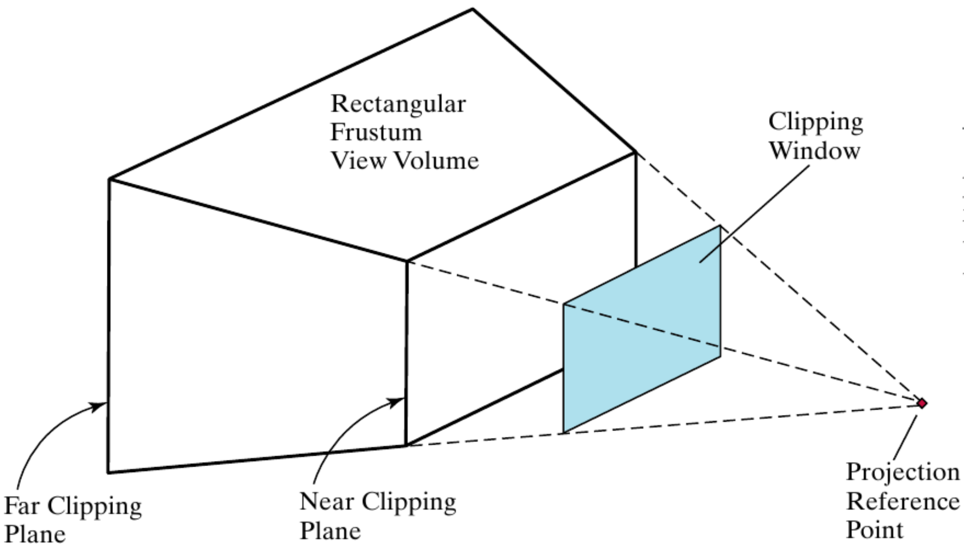
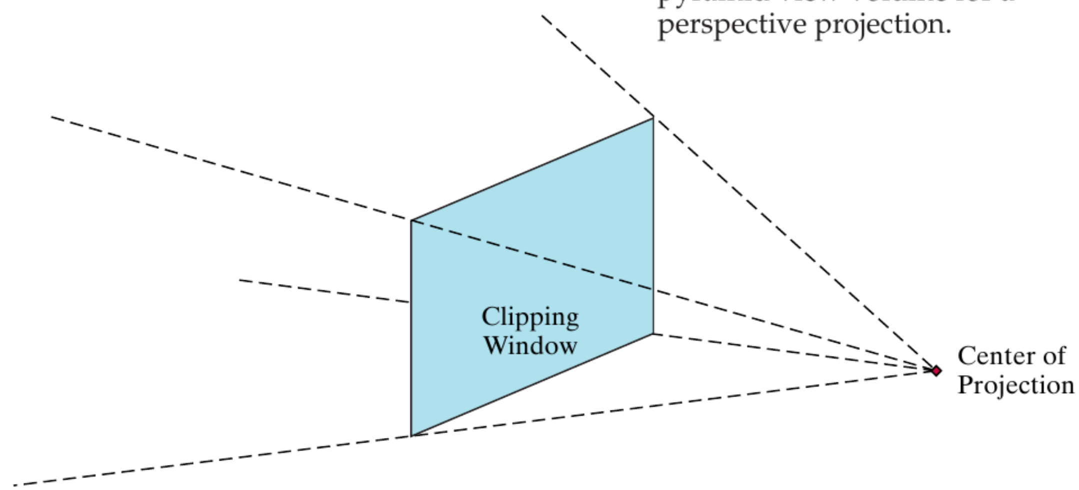


FIGURE 7–46 A perspective-projection frustum view volume with the view plane “in front” of the near clipping plane.

FIGURE 7–45 An infinite, pyramid view volume for a perspective projection.



3D Perspective Projection

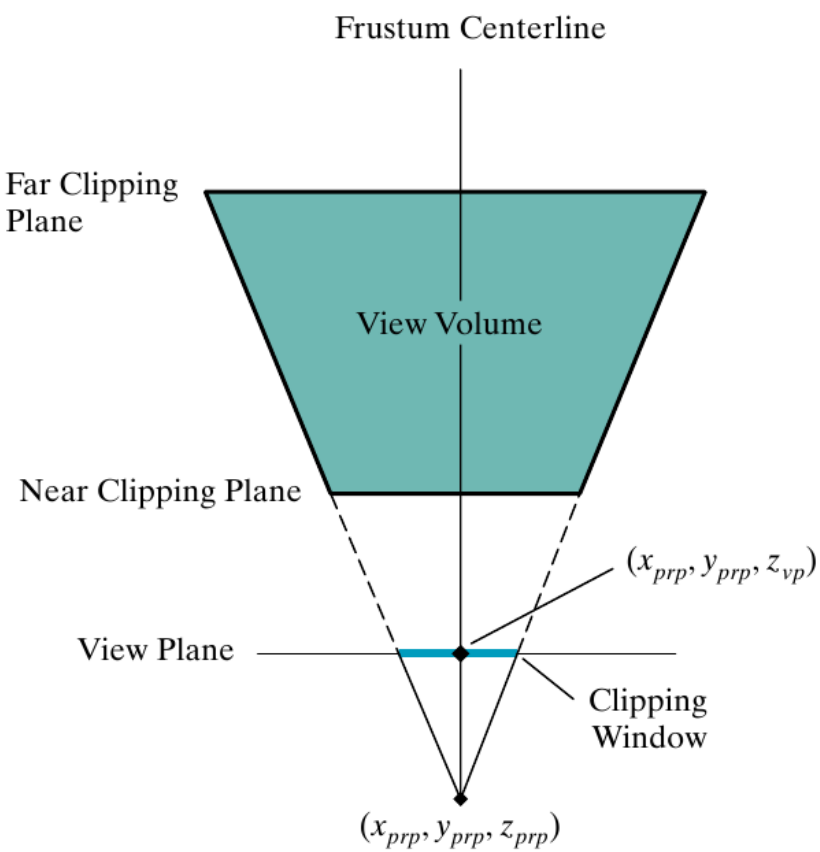


FIGURE 7-47 A symmetric perspective-projection frustum view volume, with the view plane between the projection reference point and the near clipping plane. This frustum is symmetric about its centerline when viewed from above, below, or either side.

$$xw_{\min} = x_{prp} - \frac{\text{width}}{2}, \qquad xw_{\max} = x_{prp} + \frac{\text{width}}{2}$$

$$yw_{\min} = y_{prp} - \frac{\text{height}}{2}, \qquad yw_{\max} = y_{prp} + \frac{\text{height}}{2}$$

3D Perspective Projection

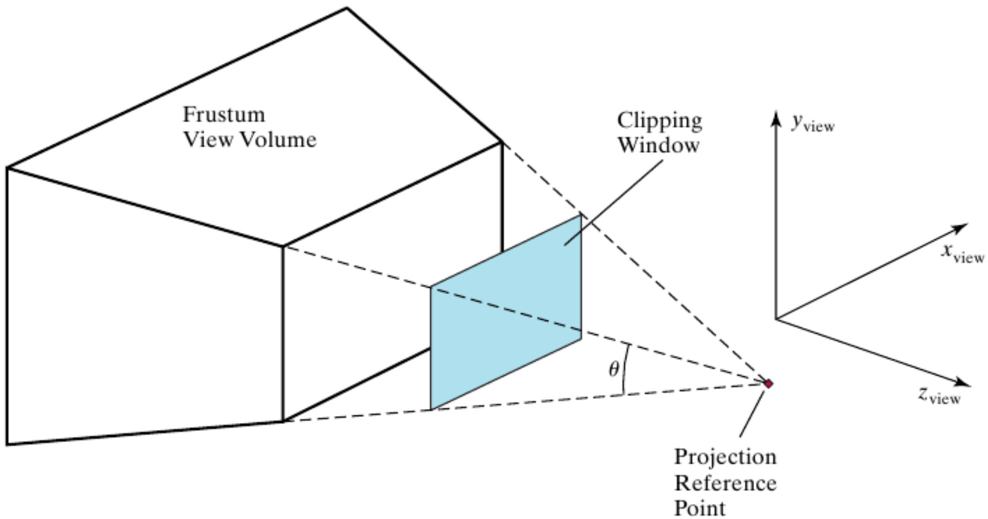


FIGURE 7-48
Field-of-view angle θ for a symmetric perspective-projection view volume, with the clipping window between the near clipping plane and the projection reference point.

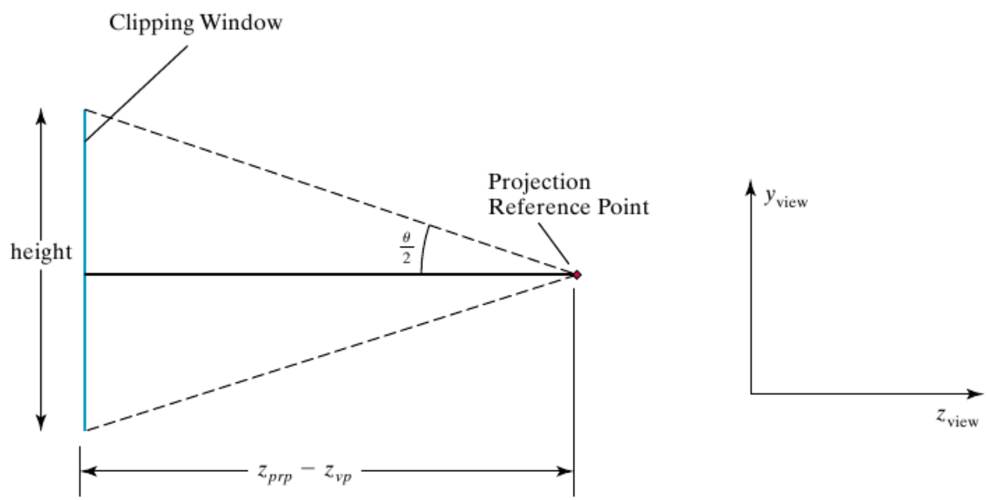


FIGURE 7-49
Relationship between the field-of-view angle θ, the height of the clipping window, and the distance between the projection reference point and the view plane.

3D Perspective Projection

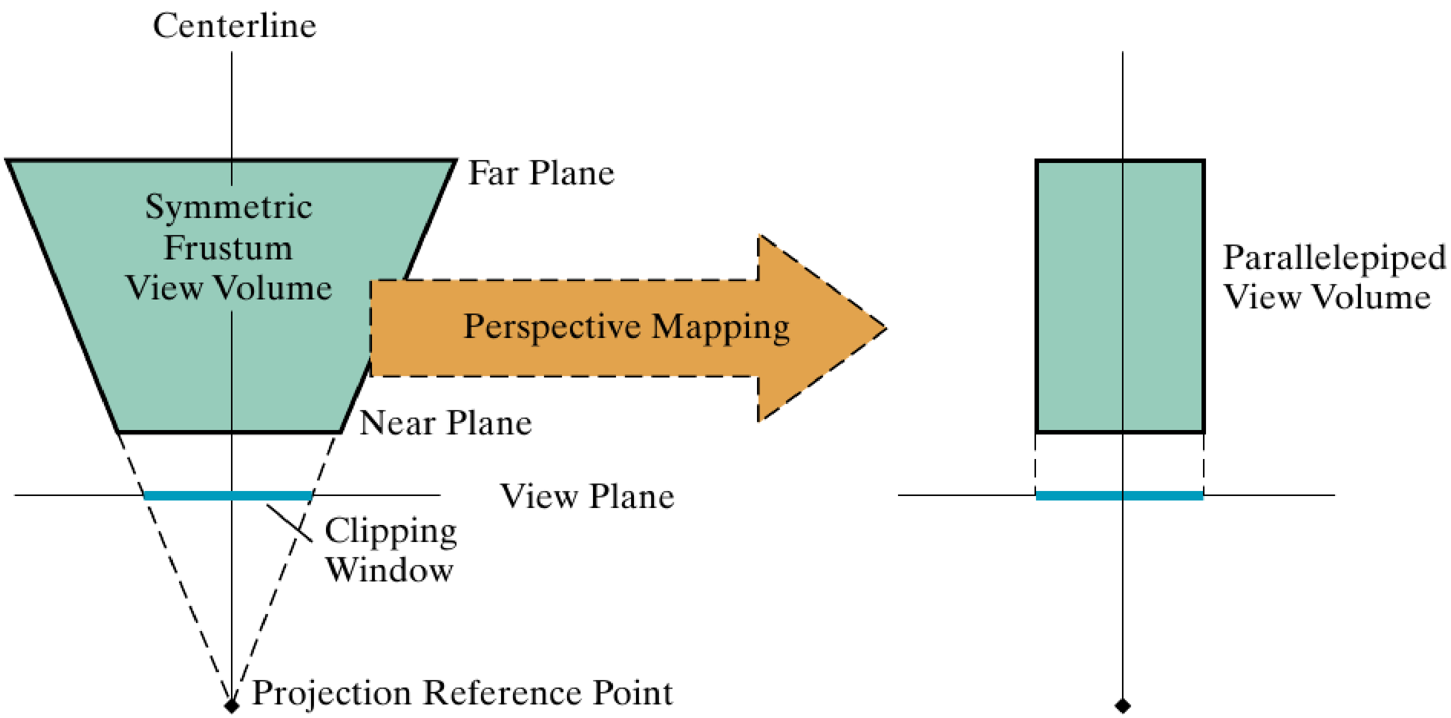


FIGURE 7-51 A symmetric frustum view volume is mapped to an orthogonal parallelepiped by a perspective-projection transformation.

3D Perspective Projection

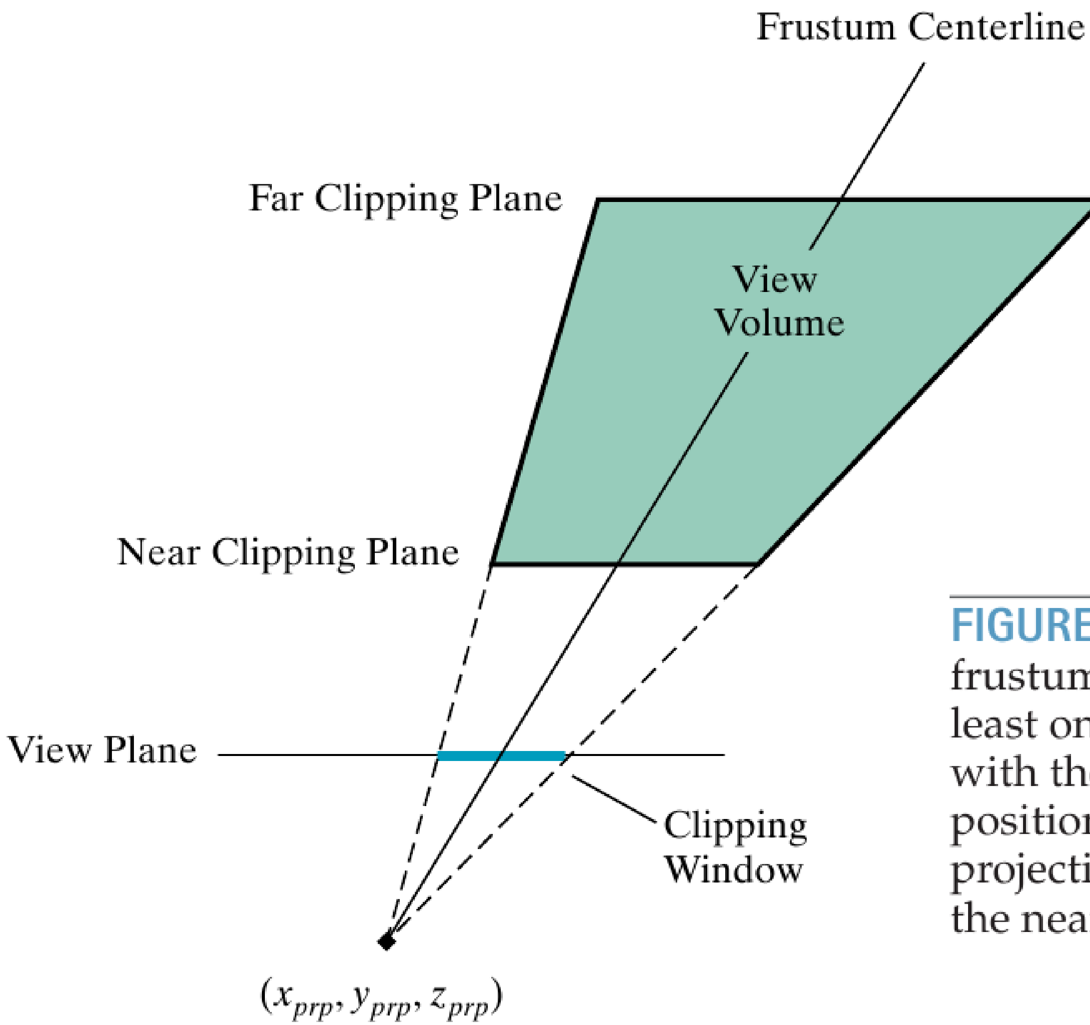


FIGURE 7-52 An oblique frustum, as viewed from at least one side or a top view, with the view plane positioned between the projection reference point and the near clipping plane.

3D Clipping Algorithms

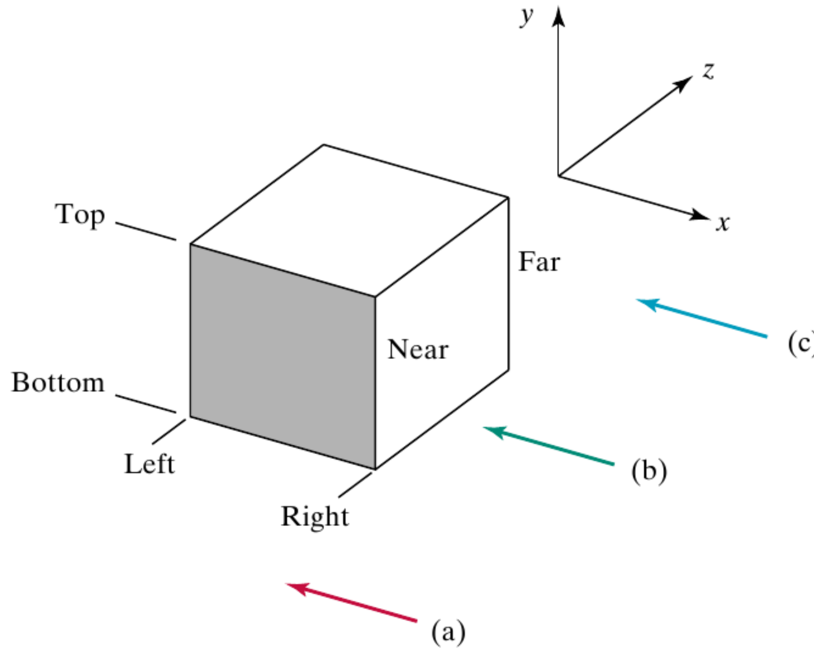


FIGURE 7-57 Values for the three-dimensional, six-bit region code that identifies spatial positions relative to the boundaries of a view volume.

011001	011000	011010
010001	010000	010010
010101	010100	010110

Region Codes
In Front of Near Plane
(a)

001001	001000	001010
000001	000000	000010
000101	000100	000110

Region Codes
Between Near and Far Planes
(b)

101001	101000	101010
100001	100000	100010
100101	100100	100110

Region Codes
Behind Far Plane
(c)

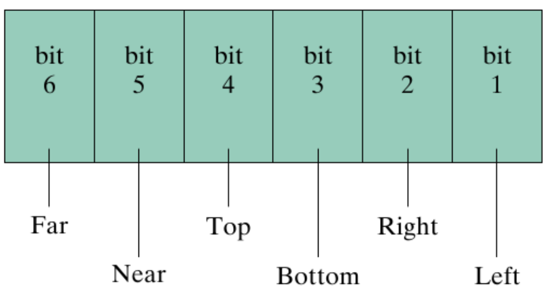


FIGURE 7-56 A possible ordering for the view-volume clipping boundaries corresponding to the region-code bit positions.

3D Clipping Algorithms

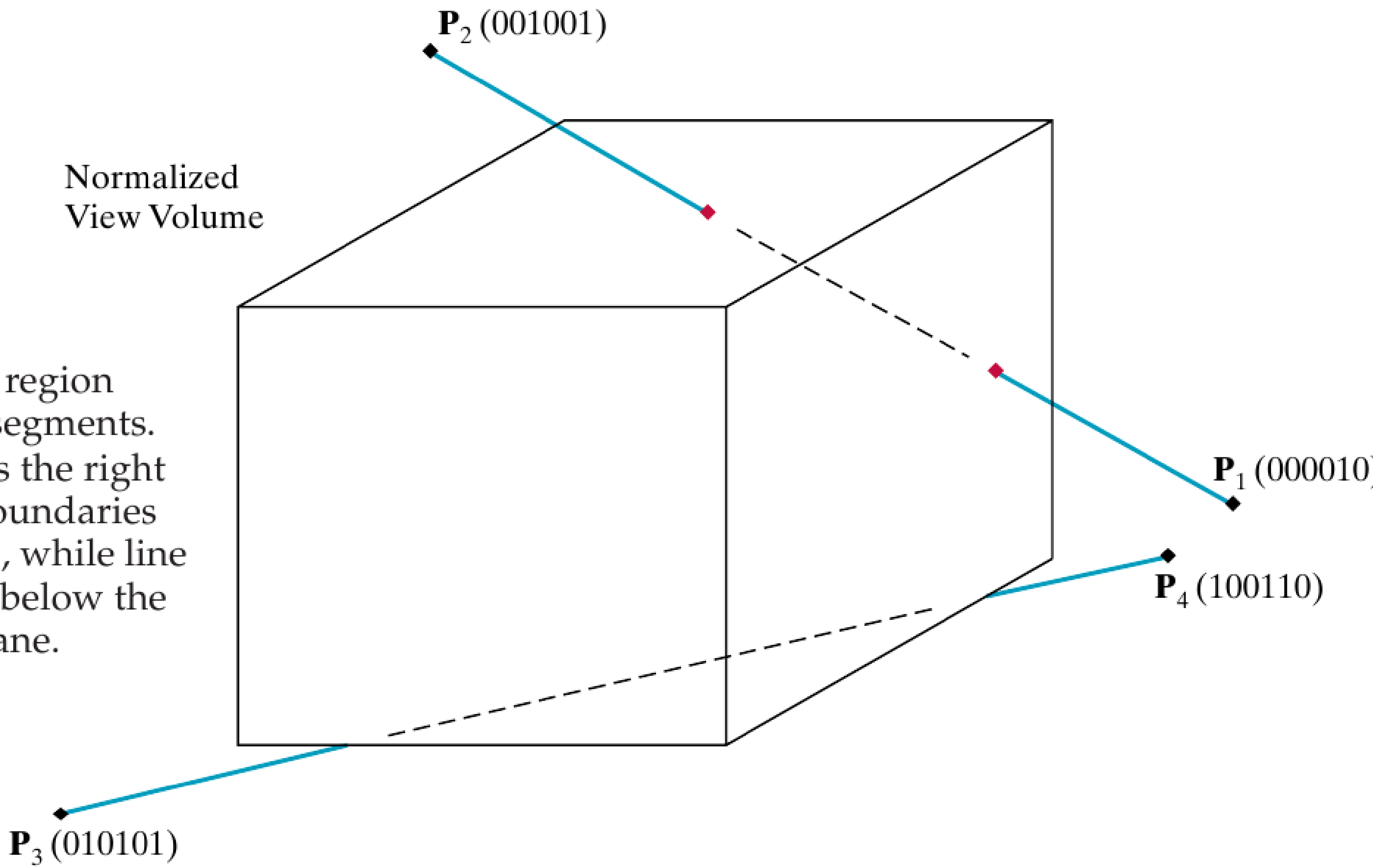


FIGURE 7-58
Three-dimensional region codes for two line segments. Line $\overline{\mathbf{P}_1\mathbf{P}_2}$ intersects the right and top clipping boundaries of the view volume, while line $\overline{\mathbf{P}_3\mathbf{P}_4}$ is completely below the bottom clipping plane.

3D Polygon Clipping

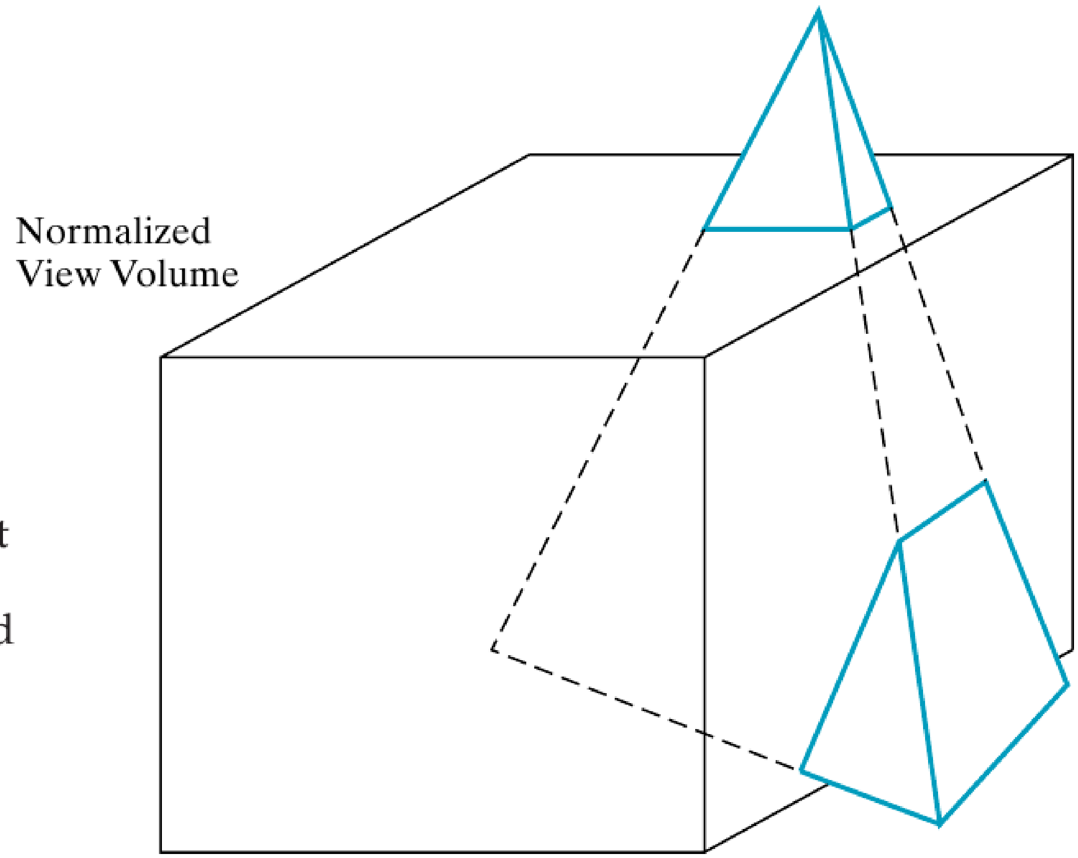


FIGURE 7-59
Three-dimensional object clipping. Surface sections that are outside the view-volume clipping planes are eliminated from the object description, and new surface facets may need to be constructed.