Implement the following algorithms:

1. A recursive bottom-up algorithm for constructing an AABB tree for an open polygonal chain $C$ (in the plane) that connects a sequence of points $p_i = (x_i, y_i)$, for $i = 0, \cdots, n$, for some $n > 0$.

2. A recursive algorithm for testing the self-intersection of the polygonal chain $C$ using the AABB tree constructed for $C$.

3. A recursive algorithm for testing the intersection between two polygonal chains $C$ and $D$ using their respective AABB trees.

We may generate a polygonal chain by sampling a cubic Bézier curve $C(t)$ at uniform parameters $t_i = i/n$, for $i = 0, \cdots, n$. 