A cubic Bézier curve $C(t) = \sum_{i=0}^{3} b_i B_3^i(t)$, $0 \leq t \leq 1$, can be approximated by a polygonal curve $L^h(t)$ connecting a sequence of curve points $C(i/2^h)$, for $i = 0, \ldots, 2^h$, within an approximation error bound (Filip et al., CAGD 1986):

$$\|C(t) - L^h(t)\| \leq \frac{3}{4} \cdot \frac{1}{4^h} \cdot \max (\|b_0 - 2b_1 + b_2\|, \|b_1 - 2b_2 + b_3\|) = \epsilon_h.$$  

More precisely, each line segment $L^h_i(t)$, $(t^h_{i-1} \leq t \leq t^h_i)$, approximates the corresponding curve segment $C^h_i(t) = C(t)$, $(t^h_{i-1} \leq t \leq t^h_i)$, within the error bound $\epsilon_h \geq 0$.

Design an interactive system that can edit an infinite line $ax + by + c = 0$ by controlling two points $p_0$ and $p_1$ on the line, and the cubic Bézier curve $C(t)$ by dragging the four control points $b_i$. Moreover, implement a recursive algorithm for computing the intersection between the cubic Bézier curve $C(t)$ and the infinite line $ax + by + c = 0$.

1. Design a recursive algorithm for computing the signed distances $d^h_{i-1}$ and $d^h_i$ of the endpoints of $L^h_i(t)$ from the line $ax + by + c = 0$.

   (a) If the two signed distances are both less than $-\epsilon_h$ or both larger than $\epsilon_h$ (i.e., $[(d^h_{i-1} < -\epsilon_h) \land (d^h_i < -\epsilon_h)]$ or $[(d^h_{i-1} > \epsilon_h) \land (d^h_i > \epsilon_h)]$), there will be no intersection between $C^h_i(t)$ and the line $ax + by + c = 0$.

   (b) Otherwise, go down to the next level of the recursive approximation by evaluating the curve at the midpoint $C(\frac{1}{2}(\frac{i-1}{2^h} + \frac{i}{2^h})) = C^h_i(\frac{t^h_{i-1} + t^h_i}{2}) = C^h_{2i} (t^h_{2i-1})$.

2. Repeat the same procedure recursively until the maximum level $h = 10$ and report an appropriate point of the correspondig line segment $L^h_i(t)$ as an approximate intersection point.

3. Each time you modify the curve $C(t)$ or the line $ax + by + c = 0$, recompute the intersection points.

4. To speed up the curve evaluation $C(t_i)$, it is important to precompute the basic functions $B_3^l(t_i)$, for $l = 0, 1, 2, 3$, at all finest parameter values $t_i = i/2^{10}$, $i = 0, \ldots, 2^{10}$. Note that these function values are fixed.