Quiz #2 (CSE4190.667)

April 11, 2012 (Monday)

Name: ____________ **Dept:** ____________ **ID No:** ____________

1. (10 points) Construct a Coons patch $\mathbf{x}(u, v)$, $0 \le u, v \le 1$, that interpolates the four boundary curves of the bivariate surface $S(u, v) = \begin{bmatrix} u \\ v \\ uv \end{bmatrix}$, for $0 \le u, v \le 1$.

$$\mathbf{x}(u,v) = (1-u)S(0,v) + uS(1,v) + (1-v)S(u,0) + vS(u,1)$$

$$- \begin{bmatrix} (1-u) & u \end{bmatrix} \begin{bmatrix} S(0,0) & S(0,1) \\ S(1,0) & S(1,1) \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix}$$

$$= (1-u)\begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix} + u\begin{bmatrix} 1 \\ v \\ v \end{bmatrix} + (1-v)\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} + v\begin{bmatrix} u \\ 1 \\ u \end{bmatrix}$$

$$- \begin{bmatrix} (1-u) & u \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix}$$

$$= \begin{bmatrix} u \\ v \\ uv \end{bmatrix} + \begin{bmatrix} u \\ v \\ uv \end{bmatrix} - \begin{bmatrix} \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} u \\ 1 \\ u \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix}$$

$$= \begin{bmatrix} u \\ v \\ uv \end{bmatrix} + \begin{bmatrix} u \\ v \\ uv \end{bmatrix} - \begin{bmatrix} u \\ v \\ uv \end{bmatrix} = \begin{bmatrix} u \\ v \\ uv \end{bmatrix}$$

2. (15 points) What are the control points $\mathbf{b}_{ij}$, $(i, j = 0, 1, 2, 3)$, for a bicubic Bézier patch $\mathbf{x}(u, v) = \begin{bmatrix} u \\ v^2 \\ uv \end{bmatrix}$ over the rectangular domain $0 \le u, v \le 1$?

$$u = \sum_{i=0}^{3} \frac{i}{3} B_i^3(u) = \sum_{i=0}^{3} \sum_{j=0}^{3} \frac{i}{3} B_i^3(u) B_j^3(v)$$

$$v^2 = v^2 \left[(1-v) + v \right] = \frac{1}{3} B_2^3(v) + B_3^3(v)$$

$$= \sum_{i=0}^{3} \frac{1}{3} B_i^3(u) B_2^3(v) + \sum_{i=0}^{3} B_i^3(u) B_3^3(v)$$

$$uv = \sum_{i=0}^{3} \sum_{j=0}^{3} \frac{ij}{9} B_i^3(u) B_j^3(v)$$

$$\mathbf{b}_{ij} = \begin{bmatrix} i/3 \\ 0 \\ ij/9 \end{bmatrix}, \quad i = 0, 1, 2, 3; \; j = 0, 1$$

$$\mathbf{b}_{i2} = \begin{bmatrix} i/3 \\ 1/3 \\ 2i/9 \end{bmatrix}, \quad i = 0, 1, 2, 3$$

$$\mathbf{b}_{i3} = \begin{bmatrix} i/3 \\ 1 \\ i/3 \end{bmatrix}, \quad i = 0, 1, 2, 3$$