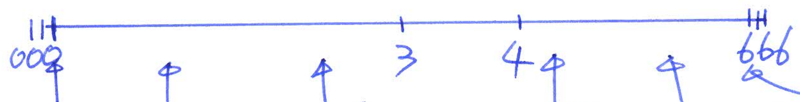
Geometric Modeling (CSE 4190.667)

Given a knot sequence $0, 0, 0, 3, 4, 6, 6, 6$ for a cubic B-spline curve $\mathbf{x}(u) = (u, N_3^3(u))$, $0 \le u \le 6$,

1. What are the B-spline control points $\mathbf{d}_i$ for the cubic curve $\mathbf{x}(u)$?
2. Using the de Boor algorithm, evaluate the function value $\mathbf{x}(5)$.
3. Using the 2-stage de Boor algorithm, evaluate the first derivative $\mathbf{x}'(5)$.
4. Using the 2-stage de Boor algorithm, evaluate the second derivative $\mathbf{x}''(5)$.

①



$\xi_0=0,\ \xi_1=1,\ \xi_2=\frac{7}{3},\ \xi_3=\frac{13}{3},\ \xi_4=\frac{16}{3},\ \xi_5=6$

$\mathbf{d}_0=\begin{bmatrix}0\\0\end{bmatrix},\ \mathbf{d}_1=\begin{bmatrix}1\\0\end{bmatrix},\ \mathbf{d}_2=\begin{bmatrix}7/3\\0\end{bmatrix},\ \mathbf{d}_3=\begin{bmatrix}13/3\\1\end{bmatrix},\ \mathbf{d}_4=\begin{bmatrix}16/3\\0\end{bmatrix},\ \mathbf{d}_5=\begin{bmatrix}6\\0\end{bmatrix},$

② $\mathbf{d}_0$

$\mathbf{d}_1$

$\mathbf{d}_2$ $\quad \mathbf{d}_3^1=\begin{bmatrix}4\\5/6\end{bmatrix}$ $\quad \mathbf{d}_4^2=\begin{bmatrix}14/3\\1/2\end{bmatrix}$ $\quad \mathbf{d}_5^3=\begin{bmatrix}15/3\\1/3\end{bmatrix}$

$\mathbf{d}_3$ $\quad \mathbf{d}_4^1=\begin{bmatrix}5\\1/3\end{bmatrix}$ $\quad \mathbf{d}_5^2=\begin{bmatrix}16/3\\1/6\end{bmatrix}$

$\mathbf{d}_4$ $\quad \mathbf{d}_5^1=\begin{bmatrix}17/3\\0\end{bmatrix}$

$\mathbf{d}_5$

③

$$\mathbf{d}_5^3=\frac{3}{2}\begin{bmatrix}2/3\\-2/6\end{bmatrix}=\begin{bmatrix}1\\-1/2\end{bmatrix}$$

④

$$\mathbf{d}_4^2=\frac{2}{3}\begin{bmatrix}1\\-1/2\end{bmatrix}=\begin{bmatrix}2/3\\-1/3\end{bmatrix}\qquad \mathbf{d}_5^3=\frac{3}{2}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\mathbf{d}_5^2=\frac{2}{2}\begin{bmatrix}2/3\\-1/3\end{bmatrix}=\begin{bmatrix}2/3\\-1/3\end{bmatrix}$$