1. $f(t) = \|\mathbf{x}(t)\|^2 = (x(t))^2 + (y(t))^2$

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$x(t) = x_0(1-t)^2 + x_1\, 2t(1-t) + x_2 t^2$

$$
\begin{aligned}
(x(t))^2 &= x_0^2(1-t)^4 + x_1^2 \cdot 4t^2(1-t)^2 + x_2^2 t^4 \\
&\quad + 2x_0x_1 \cdot 2t(1-t)^3 + 2x_0x_2 t^2(1-t)^2 + 2x_1x_2 \cdot 2t^3(1-t) \\
&= x_0^2(1-t)^4 + x_0x_1 \cdot 4t(1-t)^3 + \left(\tfrac{2}{3}x_1^2 + \tfrac{1}{3}x_0x_2\right) \cdot 6t^2(1-t)^2 \\
&\quad + x_1x_2 \cdot 4t^3(1-t) + x_2^2 t^4
\end{aligned}
$$

$$
\begin{aligned}
\therefore f(t) &= \underline{(x_0^2 + y_0^2)}\, B^4_0(t) + (\underline{x_0x_1 + y_0y_1})\, B^4_1(t) \\
&\quad + \underline{\left(\tfrac{2}{3}(x_1^2 + y_1^2) + \tfrac{1}{3}(x_0x_2 + y_0y_2)\right)} B^4_2(t) \\
&\quad + (\underline{x_1x_2 + y_1y_2})\, B^4_3(t) + (\underline{x_2^2 + y_2^2})\, B^4_4(t)
\end{aligned}
$$

$\Rightarrow$ 14 multiplications, 7 additions

12: $x_ix_j$, 2: $\times\frac{2}{3}$, $\times\frac{1}{3}$

2. (1) $f'(t) = 2\left(x(t)\cdot x'(t) + y(t)\cdot y'(t)\right) = 2g(t)$

5

위의 다섯 control point들 사이의 차이를 계산하여 네 배를 한 뒤 반으로 나누면 (결국 두 배를 하면)
$g(t)$의 네 control point를 구할 수 있다.

$\Rightarrow$ 14(+4) multiplications, $7+4=11$ additions

×2 뺄셈

(2) $g(t) = x(t)\cdot x'(t) + y(t)\cdot y'(t)$.

$$
\begin{aligned}
x(t)\cdot x'(t) &= \left(x_0(1-t)^2 + x_1\, 2t(1-t) + x_2 t^2\right)\cdot\left(2(-x_0+x_1)(1-t) + 2(-x_1+x_2)t\right) \\
&= 2x_0(-x_0+1)(1-t)^3 + \left(2x_0(-x_1+x_2) + 4x_1(-x_0+x_1)\right)t(1-t)^2 \\
&\quad + \left(4x_1(-x_1+x_2) + 2x_2(-x_0+x_1)\right)t^2(1-t) + 2x_2(-x_1+x_2)t^3
\end{aligned}
$$

$$
\begin{aligned}
\therefore g(t) &= 2\left(x_0(-x_0+x_1) + y_0(-y_0+y_1)\right)B^3_0(t) \\
&\quad + \left(\tfrac{2}{3}\left(x_0(-x_1+x_2) + y_0(-y_1+y_2)\right) + \tfrac{4}{3}\left(x_1(-x_0+x_1) + y_1(-y_0+y_1)\right)\right)B^3_1(t) \\
&\quad + \left(\tfrac{4}{3}\left(x_1(-x_1+x_2) + y_1(-y_1+y_2)\right) + \tfrac{2}{3}\left(x_2(-x_0+x_1) + y_2(-y_0+y_1)\right)\right)B^3_2(t) \\
&\quad + 2\left(x_2(-x_1+x_2) + y_2(-y_1+y_2)\right)B^3_3(t)
\end{aligned}
$$

$\Rightarrow$ 16(+2) multiplications, 12 additions

×2

12: $x_i(-x_j+x_k)$, 4: $\times\frac{2}{3}$, $\times\frac{1}{3}$

(3) $g(t)$를 계산하는 데에는 (1)이 덧셈 한 번이 더 적고
$\frac{1}{2}g(t)$를 계산한다면 (1)이 덧셈 한 번과 곱셈 두 번이 더 적다.

3.

5

$$\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}0.5\\0\\0\end{pmatrix}\begin{pmatrix}1.5\\0\\0\end{pmatrix},\ \begin{pmatrix}0.5\\1\\1\end{pmatrix}\begin{pmatrix}1.5\\1\\1\end{pmatrix},\ \begin{pmatrix}0.5\\2\\2\end{pmatrix}\begin{pmatrix}1.5\\2\\2\end{pmatrix}$$

$$\begin{pmatrix}0\\2\\2\end{pmatrix}\begin{pmatrix}1\\2\\2\end{pmatrix}\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix}0.5\\0.5\\0.5\end{pmatrix}\begin{pmatrix}1.5\\0.5\\0.5\end{pmatrix}$$

$$\begin{pmatrix}0.5\\1.5\\1.5\end{pmatrix}\begin{pmatrix}1.5\\1.5\\1.5\end{pmatrix}$$

$$\Downarrow \qquad \Downarrow$$

$$\begin{pmatrix}1\\0.5\\0.5\end{pmatrix},\ \begin{pmatrix}1\\1.5\\1.5\end{pmatrix} \qquad \begin{pmatrix}0.5\\1\\1\end{pmatrix}\begin{pmatrix}1.5\\1\\1\end{pmatrix}$$

$$X_u = 2\begin{pmatrix}1-1\\1.5-0.5\\1.5-0.5\end{pmatrix} = \begin{pmatrix}0\\2\\2\end{pmatrix} \qquad X_v = 2\begin{pmatrix}1.5-0.5\\1-1\\1-1\end{pmatrix} = \begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$X_u \wedge X_v = \begin{pmatrix}0\\4\\-4\end{pmatrix},\quad n = \begin{pmatrix}0\\\frac{\sqrt{2}}{2}\\-\frac{\sqrt{2}}{2}\end{pmatrix}$$