HW #1 (CSE4190.667)

Due: April 4, 2012 (Wednesday)

1. Given a planar quadratic Bézier curve

$$\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} B_0^2(t) + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} B_1^2(t) + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} B_2^2(t),$$

   construct an explicit Bézier form for the quartic function $f(t) = \|\mathbf{x}(t)\|^2$. What is the operation count for this construction?

2. Using the quartic Bézier function: $f(t) = \|\mathbf{x}(t)\|^2 = \sum_{i=0}^{4} f_i B_i^4(t)$, construct an explicit Bézier form for the cubic function $g(t) = \langle \mathbf{x}(t), \mathbf{x}'(t) \rangle$. What is the operation count for this construction? Compare the operation count with that for a direct construction of $g(t) = \langle \mathbf{x}(t), \mathbf{x}'(t) \rangle$ from the scratch.

3. Using the 3-stage de Casteljau evaluation method, compute $\mathbf{x}_u(0.5, 0.5), \mathbf{x}_v(0.5, 0.5)$, and $\mathbf{n}(0.5, 0.5)$ for the Bézier patch defined by the following control points:

$$\begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \end{bmatrix}$$