# Programming #3: Part II (4190.410)

## Due: October 14, 2013

A cubic Bézier curve $C(t) = \sum_{l=0}^{3} \mathbf{b}_l B_l^3(t)$, $0 \le t \le 1$, can be approximated by a polygonal curve $L^h(t)$ connecting a sequence of curve points $C(t_i^h) = C(i/2^h)$, for $i = 0, \cdots, 2^h$, within an approximation error bound (Filip et al., CAGD 1986):

$$\|C(t) - L^h(t)\| \le \frac{3}{4} \cdot \frac{1}{4^h} \cdot \max\left(\|\mathbf{b}_0 - 2\mathbf{b}_1 + \mathbf{b}_2\|, \|\mathbf{b}_1 - 2\mathbf{b}_2 + \mathbf{b}_3\|\right) = \epsilon_h.$$

More precisely, each line segment $L_i^h(t)$, $(t_{i-1}^h \le t \le t_i^h)$, approximates the corresponding curve segment $C_i^h(t) = C(t)$, $(t_{i-1}^h \le t \le t_i^h)$, within the error bound $\epsilon_h \ge 0$.

**Part I:** Design an interactive system that can show the BVH structure (i.e., the AABB tree and the LSS tree) for the Bézier curve $C(t)$, $0 \le t \le 1$. Display the rounded AABB bounding volumes generated by expanding the AABB containing each line segment $L_i^h(t)$ by $\epsilon_h$. Moreover, display the LSS bounding volumes generated by sweeping a disc of radious $\epsilon_h$ along each line segment $L_i^h(t)$, for the levels $h = 2, \cdots 10$.

**Part II:** Design an interactive system that can control the position of a query point $\mathbf{Q}$ and the shape of $C(t)$ by dragging the four control points $\mathbf{b}_l$. Moreover, implement a recursive algorithm for computing the projection line from $\mathbf{Q}$ to the nearest point on the curve $C(t)$. Display the bounding volumes that have been used in the search for the nearest point $C(\hat{t})$ by the recursive algorithm.