1. (5 points) What is the perspective projection of a point \( p = (3, 5, 7) \) from the view point \( v = (1, 2, 3) \) onto the line \( x + y + z + 1 = 0 \)?

\[
\hat{p} = (3, 5, 7, 1) \\
\hat{v} = (1, 2, 3, 1) \\
\hat{m} = (1, 1, 1, 1) \\
\hat{m} \times (\hat{p} \times \hat{v}) = \langle \hat{m}, \hat{v} \rangle \hat{p} - \langle \hat{m}, \hat{p} \rangle \hat{v} \\
= 7 \hat{p} - 16 \hat{v} \\
= (5, 3, 1, -9) \\
= \left(-\frac{5}{9}, -\frac{3}{9}, -\frac{1}{9}, 1\right)
\]
2. (7 points) Consider two parallel planes:

\[ \Pi_1 : \ ax + by + cz + d_1 = 0, \]
\[ \Pi_2 : \ ax + by + cz + d_2 = 0. \]

What is the affine transformation from \( \mathbb{R}^3 \) to \( \mathbb{R}^1 \) that sends \( \Pi_1 \) to \(-1\) and \( \Pi_2 \) to \(1\)?

\[
\begin{bmatrix}
2a & 2b & 2c & d_1 + d_2 \\
0 & 0 & 0 & d_1 - d_2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
2ax + 2by + 2cz + d_1 + d_2 \\
d_1 - d_2
\end{bmatrix}
\]

For \((x, y, z) \in \Pi_1\), we have

\[ 2(ax + by + cz) = -2d_1 \]
\[ \therefore 2ax + 2by + 2cz + d_1 + d_2 = d_2 - d_1 \]

For \((x, y, z) \in \Pi_2\), we have

\[ 2(ax + by + cz) = -2d_2 \]
\[ \therefore 2ax + 2by + 2cz + d_1 + d_2 = d_1 - d_2 \]

Hence, the above affine transformation sends \( \Pi_1 \) and \( \Pi_2 \) to \(-1\) and \(1\), respectively.
3. (8 points) Using the wedge-product operation discussed in class, answer the following questions. What is the plane that is determined by three points \((1, 2, 3, 1), (3, 5, 7, 1),\) and \((2, 3, 5, 1)\)? What is its intersection with other planes \(x + y + z + 1 = 0\) and \(x - y - z + 1 = 0\)?

\[
(1, 2, 3, 1) \wedge (3, 5, 7, 1) \wedge (2, 3, 5, 1)
\]

\[
= (2, 0, -1, 1)
\]

\[
\therefore 2x - z + 1 = 0
\]

\[
(2, 0, -1, 1) \wedge (1, 1, 1, 1) \wedge (1, -1, -1, 1)
\]

\[
= (2, -2, 2, -2)
\]

\[
= (-1, 1, -1, 1)
\]

\[
\therefore (-1, 1, -1)
\]