Computer Graphics

(Comp 4190.410)

Midterm Exam: November 1, 2004

1. (15 points) We want to draw a line $L: ax + by + c = 0$ (of slope $0 < m < 1$) from x_0 to $x_n = x_0 + n$, where a, b, c, x_0 are integers. (Note that in our textbook we took $a = \Delta y$ and $b = -\Delta x$.) The corresponding y values at the two end points may not have integer values. Let $y_0 = round(-\frac{ax_0+c}{b})$ and take (x_0, y_0) as the starting point for a modified Bresenham algorithm. Then we test the sign of

$$F(x_0 + 1, y_0 + \frac{1}{2}) = 2a(x_0 + 1) + 2b(y_0 + \frac{1}{2}) + 2c = 2(ax_0 + by_0 + c) + 2a + b,$$

and may proceed as in the conventional Bresenham algorithm. However, the point (x_0, y_0) is not located on the line L; hence, $ax_0 + by_0 + c \neq 0$. Does the algorithm still work? Otherwise, explain why the modified algorithm doesn't work by giving a counter example or using counter arguments.

2. (20 points)

   (a) (10 points) Consider a rotation R_1 about an axis $(1, 0, 0)$ by angle $90°$ and another rotation R_2 about an axis $(0, 1, 0)$ by angle $120°$. What is the axis and angle of the composite rotation R_2R_1?

   (b) (10 points) Given a sequence of unit quaternions $q_1, \cdots, q_n$, the product $q_1 \cdot q_2 \cdots q_n$ represents a composite 3D rotation in the local coordinate and the product $q_n \cdot q_{n-1} \cdots q_2 \cdot q_1$ represents a composite 3D rotation in the global coordinate. Explain why this is the case.

3. (15 points) Construct a 2D perspective transformation

$$\mathbf{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

so that it sends 2D points $(0, 0, 1), (1, 0, 1), (0, 1, 1), (1, 1, 1)$ (in homogeneous coordinates) to 2D points $(0, 0, 1), (1, 0, 1), (0, 1, 1), (2, 3, 1)$, respectively.

4. (15 points) In our textbook, the authors claim that the Nicholl-Lee-Nicholl algorithm can be applied only to two-dimensional clipping. What is the main difficulty in extending the NLN algorithm to three-dimensional clipping? How can you reduce the three-dimensional clipping problem to two subproblems of two-dimensional clipping? How can you compare the overall performance of this approach with other algorithms?

5. (40 points)

(a) (10 points) Given two parallel planes L_1 and L_2 (with $d_1 > d_2$)

$$L_1 : ax + by + cz + d_1 = 0,$$
$$L_2 : ax + by + cz + d_2 = 0,$$

set up two linear inequality conditions for clipping a 3D point $P(x, y, z)$ with respect to the infinite volume bounded by the two planes.

(b) (15 points) Construct an affine transformation that sends the plane L_1 to -1 and the other plane L_2 to 1.

(c) (15 points) Using the above result and the 3D viewing transformation based on the line geometry, discuss how to transform an arbitrary 3D view volume to a normalized view volume.