Quiz #3 (CSE 400.001)

Monday, October 11, 2010

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (10 points) Solve the following initial value problem:

$$x^3y''' + 7x^2y'' - 2xy' - 10y = 0, \quad y(1) = 1,\ y'(1) = -7, y''(1) = 44.$$

$$m(m-1)(m-2) + 7m(m-1) - 2m - 10 = 0$$

$$(m+1)(m+5)(m-2) = 0 \qquad +2$$

$$y_1 = x^2, \quad y_2 = \frac{1}{x}, \quad y_3 = \frac{1}{x^5} \qquad +2$$

$$y = c_1 x^2 + c_2 \cdot \frac{1}{x} + c_3 \cdot \frac{1}{x^5} \qquad +1$$

$$\left.\begin{aligned} y' &= 2c_1 x - c_2 \cdot \frac{1}{x^2} - 5c_3 \cdot \frac{1}{x^6} \\ y'' &= 2c_1 + 2c_2 \cdot \frac{1}{x^3} + 30c_3 \cdot \frac{1}{x^7} \end{aligned}\right] \quad +2$$

$$\begin{cases} c_1 + c_2 + c_3 = 1 \\ 2c_1 - c_2 - 5c_3 = -7 \\ 2c_1 + 2c_2 + 30c_3 = 44 \end{cases} \Rightarrow \quad \begin{aligned} c_1 &= 0 \\ c_2 &= -\frac{1}{2} \\ c_3 &= \frac{3}{2} \end{aligned} \qquad +2$$

$$\therefore y = -\frac{1}{2x} + \frac{3}{2x^5} \qquad +1$$

2. (10 points) Solve the following system of ODEs:

$$y_1' = y_1 + 2y_2 + t^2$$
$$y_2' = 2y_1 + y_2 - t^2$$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $\det(A - \lambda I) = (\lambda+1)(\lambda+3) = 0$ (+1)

$\lambda_1 = -1$, $x^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $\lambda_2 = 3$, $x^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (+1)

$y_h = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t}$ (+1)

$Y u' = \begin{bmatrix} e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} t^2 \\ -t^2 \end{bmatrix}$ (+2)

$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \frac{1}{2e^{2t}} \begin{bmatrix} e^{3t} & -e^{3t} \\ e^{-t} & e^{-t} \end{bmatrix} \begin{bmatrix} t^2 \\ -t^2 \end{bmatrix} = \begin{bmatrix} t^2 e^{t} \\ 0 \end{bmatrix}$ (+1)

$u_1 = \int_0^t \tilde{t}^2 e^{\tilde{t}} d\tilde{t} = t^2 e^t - 2te^t + 2e^t - 2$ (+2)

$u_2 = 0$ (+1)

$y_p = (t^2e^t - 2te^t + 2e^t - 2) \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2 - 2t + 2 - 2e^{-t})$

$\therefore y = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2 - 2t + 2)$

(+1)