Quiz #4 (CSE 400.001)

November 21, 2012 (Wednesday)

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (5 points) Compute the following integral numerically using the Gauss quadrature with $n = 3$:

$$\int_1^3 \frac{\sin^2 x}{x} dx$$

Solution:

Let $x = t + 2$, then $dx = dt$ and $\int_1^3 \frac{\sin^2 x}{x} dx = \int_{-1}^1 \frac{\sin^2(t+2)}{t+2} dt$.

$\int_{-1}^1 \frac{\sin^2(t+2)}{t+2} dt \approx 0.55556 * \frac{\sin^2(2-0.77460)}{(2-0.77460)} + 0.88889 * \frac{\sin^2 2}{2} + 0.55556 * \frac{\sin^2(2+0.77460)}{(2+0.77460)}$

2. (10 points) The function $f(x) = e^x - 2$ has exactly one zero between 0 and 1 since $f(0)f(1) < 0$, while $f'(x) > 0$ on $[0, 1]$. When the Newton method starts at $x_0 = 1$, how many iterations are needed to produce the solution to 5D accuracy? Show the details of your work.

Solution:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{e-2}{e} = \frac{2}{e}$$

$$\frac{f''(s)}{2f'(s)} \approx \frac{f''(x_1)}{2f'(x_1)} = \frac{e^{x_1}}{2e^{x_1}} = \frac{1}{2} = 0.5$$

$$|\epsilon_n| \approx 0.5 \cdot \epsilon_{n-1}^2 \approx 0.5^{2^n - 1} \cdot \epsilon_0^{2^n} \leq 5 \cdot 10^{-6}$$

$$\epsilon_1 - \epsilon_0 = (\epsilon_1 - s) - (\epsilon_0 - s) = -x_1 + x_0 = 1 - \frac{2}{e}$$

$$\epsilon_1 = \epsilon_0 + 1 - \frac{2}{e} \approx 0.5\epsilon_0^2$$

$$\epsilon_0^2 - 2\epsilon_0 + \frac{4}{e} - 2 \approx 0$$

$$\epsilon_0 \approx 1 - \sqrt{3 - \frac{4}{e}} \approx -0.24$$

$$n = 2: \quad 0.5^3 \cdot 0.24^4 \approx 0.0004 > 5 \cdot 10^{-6}$$
$$n = 3: \quad 0.5^7 \cdot 0.24^8 \approx 0.00000009 < 5 \cdot 10^{-6}$$

Hence, $n = 3$ iterations are necessary.