Quiz #3 (CSE 400.001)

Monday, October 7, 2013

Name: ____________________ **E-mail:** ____________________

Dept: ____________________ **ID No:** ____________________

1. (5 points) Solve the following equation using the Power Series Method:

$$(1-x)y' - y = 0.$$

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$y' = \sum_{m=1}^{\infty} m a_m x^{m-1} = \sum_{s=0}^{\infty} (s+1) a_{s+1} x^s$$

+1

$$(1-x)y' - y = y' - xy' - y$$

$$= \sum_{s=0}^{\infty} (s+1) a_{s+1} x^s - \sum_{s=1}^{\infty} s a_s x^s - \sum_{s=0}^{\infty} a_s x^s$$

$$= 0$$

+2

$$a_1 - a_0 + \sum_{s=1}^{\infty} \left[(s+1) a_{s+1} - (s+1) a_s \right] x^s = 0$$

$$\therefore a_1 = a_0$$

$$a_{s+1} = a_s \quad \text{for } s = 1, 2, 3, \cdots$$

+1

$$\therefore y = \sum_{m=0}^{\infty} a_0 x^m = a_0 \sum_{m=0}^{\infty} x^m = \frac{a_0}{1-x}$$

+1

$$(|x| < 1)$$

2. (15 points) Solve the following initial value problem:

$$\begin{aligned} y_1' &= -2y_1 + y_2 + 2e^{-t}, \quad & y_1(0) = 2, \\ y_2' &= y_1 - 2y_2 + 3t, \quad & y_2(0) = 0. \end{aligned}$$

$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$, $\det(A - \lambda I) = (\lambda+2)^2 - 1 = 0$ (+1)

$\lambda_1 = -3$, $x^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $\lambda_2 = -1$, $x^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (+2)

$\Psi u' = \begin{bmatrix} e^{-3t} & e^{-t} \\ -e^{-3t} & e^{-t} \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix} = g$ (+2)

$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \frac{1}{2} e^{4t} \begin{bmatrix} e^{-t} & -e^{-t} \\ e^{-3t} & e^{-3t} \end{bmatrix} \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix} = \begin{bmatrix} e^{2t} - \frac{3}{2} t e^{3t} \\ 1 + \frac{3}{2} t e^{t} \end{bmatrix}$ (+2)

$u_1 = \frac{1}{2} e^{2t} + \frac{1}{6} e^{3t} - \frac{1}{2} t e^{3t} + c_1$ (+2)

$u_2 = t - \frac{3}{2} e^{t} + \frac{3}{2} t e^{t} + c_2$

$y = c_1 \begin{bmatrix} e^{-3t} \\ -e^{-3t} \end{bmatrix} + c_2 \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} e^{-t} + t e^{-t} + t - \frac{4}{3} \\ -\frac{1}{2} e^{-t} + t e^{-t} + 2t - \frac{5}{3} \end{bmatrix}$ (+2)

$y(0) = \begin{bmatrix} c_1 + c_2 - \frac{5}{6} \\ -c_1 + c_2 - \frac{13}{6} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (+2)

$\therefore c_1 = \frac{1}{3}$, $c_2 = \frac{5}{2}$

$y = \frac{1}{3} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} t e^{-t} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} t - \frac{1}{3} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(+2)