Quiz #2 (CSE4190.410)

September 28, 2011 (Wednesday)

Name: ____________ **Dept:** ____________ **ID No:** ____________

1. (5 points) What is the perspective projection of a point $\mathbf{p} = (3, 5, 7)$ from the view point $\mathbf{v} = (1, 2, 3)$ onto the line $x + y + z + 1 = 0$?

$$\hat{\mathbb{p}} = (3, 5, 7, 1)$$

$$\hat{\mathbb{v}} = (1, 2, 3, 1)$$

$$\hat{\mathbb{n}} = (1, 1, 1, 1)$$

$$\hat{\mathbb{n}} \times (\hat{\mathbb{p}} \times \hat{\mathbb{v}}) = \langle \hat{\mathbb{n}}, \hat{\mathbb{v}} \rangle \hat{\mathbb{p}} - \langle \hat{\mathbb{n}}, \hat{\mathbb{p}} \rangle \hat{\mathbb{v}}$$

$$= 7\hat{\mathbb{p}} - 16\hat{\mathbb{v}}$$

$$= (5, 3, 1, -9)$$

$$= \left(-\frac{5}{9}, -\frac{3}{9}, -\frac{1}{9}, 1\right)$$

$$\therefore \left(-\frac{5}{9}, -\frac{3}{9}, -\frac{1}{9}\right)$$

2. (7 points) Consider two parallel planes:

$$\Pi_1: \quad ax + by + cz + d_1 = 0,$$
$$\Pi_2: \quad ax + by + cz + d_2 = 0.$$

What is the affine transformation from R^3 to R^1 that sends Π_1 to -1 and Π_2 to 1?

$$\begin{bmatrix} 2a & 2b & 2c & d_1+d_2 \\ 0 & 0 & 0 & d_1-d_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2ax+2by+2cz+d_1+d_2 \\ d_1-d_2 \end{bmatrix}$$

For $(x,y,z) \in \Pi_1$, we have

$$2(ax+by+cz) = -2d_1$$

$$\therefore 2ax+2by+2cz+d_1+d_2 = d_2-d_1$$

For $(x,y,z) \in \Pi_2$, we have

$$2(ax+by+cz) = -2d_2$$

$$\therefore 2ax+2by+2cz+d_1+d_2 = d_1-d_2$$

Hence, the above affine transformation sends Π_1 and Π_2 to -1 and 1, respectively.

3. (8 points) Using the wedge-product operation discussed in class, answer the following questions. What is the plane that is determined by three points $(1, 2, 3)$, $(3, 5, 7)$, and $(2, 3, 5)$? What is its intersection with other planes $x + y + z + 1 = 0$ and $x - y - z + 1 = 0$?

$$(1,2,3,1) \wedge (3,5,7,1) \wedge (2,3,5,1)$$

$$= (2,0,-1,1)$$

$$\therefore 2x - z + 1 = 0$$

$$(2,0,-1,1) \wedge (1,1,1,1) \wedge (1,-1,-1,1)$$

$$= (2,-2,2,-2)$$

$$= (-1,1,-1,1)$$

$$\therefore (-1,1,-1)$$