Quiz #2 (CSE4190.410)

September 25, 2013 (Wednesday)

1. (10 points) Consider three parallel planes:

$$\Pi_i : \quad ax + by + cz + d_i = 0, \quad (i = 0, 1, 2).$$

   (a) (1 points) What is the affine transformation from R^3 to R^1 that sends Π_0 to d_0, Π_1 to d_1, and Π_2 to d_2?

   (b) (1 points) What is the 1D translation that sends d_0 to 0, d_1 to $\bar{d}_1 = d_1 - d_0$, and d_2 to $\bar{d}_2 = d_2 - d_0$?

   (c) (7 points) What is the 1D perspective transformation that sends 0 to 0, $\bar{d}_1$ to 1, and $\bar{d}_2$ to 2?

   (d) (1 point) What is the composite perspective transformation from R^3 to R^1 that sends Π_0 to 0, Π_1 to 1, and Π_2 to 2?

(a) $\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -d_0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} A & B \\ C & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} B \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $B = 0$ +1

$\begin{bmatrix} A & 0 \\ C & 1 \end{bmatrix}\begin{bmatrix} \bar{d}_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{A\bar{d}_1}{C\bar{d}_1 + 1} = 1 \quad \therefore A = C + \frac{1}{\bar{d}_1}$

$\begin{bmatrix} A & 0 \\ C & 1 \end{bmatrix}\begin{bmatrix} \bar{d}_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \frac{A\bar{d}_2}{C\bar{d}_2 + 1} = 2 \quad \therefore A = 2C + \frac{2}{\bar{d}_2}$ +2

$C + \frac{1}{\bar{d}_1} = 2C + \frac{2}{\bar{d}_2} \Rightarrow C = \frac{1}{\bar{d}_1} - \frac{2}{\bar{d}_2} = \frac{\bar{d}_2 - 2\bar{d}_1}{\bar{d}_1\bar{d}_2}$ +2

$A = \frac{2}{\bar{d}_1} - \frac{2}{\bar{d}_2} = \frac{2(\bar{d}_2 - \bar{d}_1)}{\bar{d}_1\bar{d}_2}$ +2

$\therefore \begin{bmatrix} 2(\bar{d}_2 - \bar{d}_1) & 0 \\ \bar{d}_2 - 2\bar{d}_1 & \bar{d}_1\bar{d}_2 \end{bmatrix}$

(d) $\begin{bmatrix} 2(\bar{d}_2 - \bar{d}_1) & 0 \\ \bar{d}_2 - 2\bar{d}_1 & \bar{d}_1\bar{d}_2 \end{bmatrix}\begin{bmatrix} 1 & -d_0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$