Computer Graphics

(Comp 4190.410)

Midterm Exam: October 31, 2012

1. (20 points)

   (a) (5 points) What are the main advantages of the raster scan system over the vector display system?

   (b) (5 points) What are the main advantages of Bresenham's line-drawing algorithm over the DDA algorithm.

   (c) (5 points) What are the main advantages of weighted area sampling over unweighted area sampling?

   (d) (5 points) How can you extend the flood-fill algorithm discussed in class to 3D?

2. (20 points) Consider two pairs of non-overlapping parallel planes:

$$\Pi_1 : \; ax + by + cz + d_1 = 0, \; \Pi_2 : \; ax + by + cz + d_2 = 0;$$

$$P_1 : \; \alpha x + \beta y + \gamma z + \delta_1 = 0, \; P_2 : \; \alpha x + \beta y + \gamma z + \delta_2 = 0.$$

   What is the geometric meaning of the following affine transformation? Explain why.

$$\begin{bmatrix} \alpha & 0 \\ \beta & 0 \\ \gamma & 0 \\ 0 & -(\alpha^2 + \beta^2 + \gamma^2) \end{bmatrix} \begin{bmatrix} 1 & \delta_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_2 - \delta_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & d_2 - d_1 \end{bmatrix} \begin{bmatrix} 1 & -d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3. (15 points) Consider a rotation R_1 about an axis $(1, 0, 0)$ by angle 60° and another rotation R_2 about an axis $(0, 1, 0)$ by angle 60°. What are the composite rotations $R_2 R_1$ and $R_1 R_2$? Answer in terms of the **axis** and **angle** of rotation instead of the 3×3 matrix representation. What is the third rotation R_3 that converts $R_2 R_1$ and $R_1 R_2$, i.e., $R_3(R_2 R_1) = R_1 R_2$? Also answer in terms of the axis and angle of the rotation R_3.

4. (15 points) Design an algorithm for testing the intersection between two line segments in the plane. The first line segment is determined by two end points $\mathbf{p}_0 = (x_0, y_0)$ and $\mathbf{p}_1 = (x_1, y_1)$, and the second one has two end points $\mathbf{p}_2 = (x_2, y_2)$ and $\mathbf{p}_3 = (x_3, y_3)$. Formulate the condition for an intersection in their interior points using the hat notation (i.e., $\hat{\mathbf{p}}$) for homogeneous coordinates, the cross or wedge product (i.e., $\times$ or $\wedge$), and the inner product (i.e., $\langle \; , \; \rangle$) as discussed in class.

5. (30 points)

   (a) (15 points) Design a data structure and a recursive bottom-up algorithm for constructing an AABB tree for an open polygonal chain $\mathcal{C}$ (in the plane) that connects a sequence of points $\mathbf{p}_i = (x_i, y_i)$, for $i = 0, \cdots, 2^k$, for some $k > 0$. To make life easy, you may split each subchain in the middle into two pieces with the same number of edges.

   (b) (15 points) Design a recursive algorithm for testing the self-intersection of the polygonal chain $\mathcal{C}$ using the AABB tree constructed in (a). For the intersection test of two edges, you may assume the result from the previous Problem #4.