Quiz #1 (CSE 400.001)

Tuesday, March 20, 2001

Name: ____________________ E-mail: ____________________

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1. (5 points) Solve

$$(3xe^y + 2y)dx + (x^2e^y + x)dy = 0$$

$P(x,y) = 3xe^y + 2y, \quad Q(x,y) = x^2e^y + x$

$R(x) = \frac{1}{Q}(P_y - Q_x) = \frac{1}{x^2e^y + x}\left(3xe^y + 1 - (2xe^y + 1)\right)$

$= \frac{1}{x}$ (+1)

$F(x) = \exp\left(\int \frac{1}{x}dx\right) = \exp(\ln x) = x$

$(3x^2e^y + 2xy)dx + (x^3e^y + x^2)dy = 0$ (+2)

$u(x,y) = \int (3x^2e^y + 2xy)dx = x^3e^y + x^2y + k(y)$ (+1)

$u_y = x^3e^y + x^2 + k'(y) = x^3e^y + x^2$

$\therefore k(y) = c_1$

$u(x,y) = x^3e^y + x^2y = c$ (+1)

2. (5 points) Solve the following initial value problem

$$y' + xy = xy^{-1}, \quad y(0) = 2.$$

$u = y^{1-(-1)} = y^2$

$u' = 2y \cdot y'$ (+2)

$2y \cdot y' + 2x \cdot y^2 = 2x$

$u' + 2x \cdot u = 2x$ (+1)

$u = e^{-\int 2x\,dx}\left[\int e^{\int 2x\,dx} \cdot 2x\,dx + c\right]$

$= e^{-x^2}\left[\int e^{x^2} \cdot 2x\,dx + c\right]$

$= e^{-x^2} \cdot \left[e^{x^2} + c\right] = c \cdot e^{-x^2} + 1$ (+1)

$4 = c + 1 \Rightarrow c = 3.$

$u = 1 + 3e^{-x^2} \Rightarrow y = \sqrt{1 + 3 \cdot e^{-x^2}}.$ (+1)

3. (5 points) Apply Picard's iteration to the following problem. Compute $y_1(x)$ and $y_2(x)$.

$$y' = y - y^2, \quad y(0) = \frac{1}{2}.$$

$f(x, y) = y - y^2, \quad x_0 = 0, \quad y_0 = \frac{1}{2}$ (+1)

$y_1 = y_0 + \int_0^x f(t, y_0)\,dt = \frac{1}{2} + \int_0^x f(t, \frac{1}{2})\,dt$

$= \frac{1}{2} + \int_0^x (\frac{1}{2} - \frac{1}{4})\,dt$

$= \frac{1}{2} + \frac{1}{4}x$ (+2)

$y_2 = y_0 + \int_0^x f(t, y_1)\,dt = \frac{1}{2} + \int_0^x f(t, \frac{1}{2} + \frac{1}{4}t)\,dt$

$= \frac{1}{2} + \int_0^x \left[(\frac{1}{2} + \frac{1}{4}t) - (\frac{1}{2} + \frac{1}{4}t)^2\right] dt$

$= \frac{1}{2} + \int_0^x (\frac{1}{4} - \frac{1}{16}t^2)\,dt$

$= \frac{1}{2} + \frac{1}{4}x - \frac{1}{48}x^3$ (+2)