Quiz #3 (CSE 400.001)

Wednesday, October 6, 2004

Name: _____ **E-mail:** _____

Dept: _____ **ID No:** _____

1. (10 points) Using the Convolution theorem, solve the following integrodifferential equation

$$y'(t) = 1 - \sin t - \int_0^t y(\tau)d\tau, \quad y(0) = 1.$$

$sY - 1 = \frac{1}{s} - \frac{1}{s^2+1} - \frac{1}{s} \cdot Y$ (+2)

$(s+\frac{1}{s})Y = 1 + \frac{1}{s} - \frac{1}{s^2+1}$

$Y = \frac{s+1}{s^2+1} - \frac{s}{(s^2+1)^2}$ (+2)

$y(t) = \cos t + \sin t - \int_0^t \sin\tau \cdot \cos(t-\tau)d\tau$ (+2)

$= \cos t + \sin t - \frac{1}{2}\int_0^t [\sin t + \sin(2\tau - t)]d\tau$

$= \cos t + \sin t - \frac{1}{2}t\sin t + \frac{1}{4}\underbrace{[\cos(2\tau - t)]_0^t}_{=0}$ (+2)

$= \cos t + (1 - \frac{1}{2}t)\sin t$ (+2)

2. (10 points) Solve the following differential equation

$$y'' + 6y' + 25y = 4\delta(t-\pi) + 4\delta(t-2\pi), \quad y(0) = 1,\ y'(0) = 1.$$

$$s^2Y - s - 1 + 6[sY - 1] + 25Y = 4e^{-\pi s} + 4e^{-2\pi s} \quad (+2)$$

$$(s^2+6s+25)Y = s + 7 + 4(e^{-\pi s} + e^{-2\pi s})$$

$$Y = \frac{s+3}{(s+3)^2+4^2} + \frac{4}{(s+3)^2+4^2} + \frac{4}{(s+3)^2+4^2}\left(e^{-\pi s} + e^{-2\pi s}\right) \quad (+2)$$

$$y(t) = e^{-3t}\cos 4t + e^{-3t}\sin 4t \quad (+2)$$
$$+ e^{-3(t-\pi)}\sin 4(t-\pi)\,u(t-\pi) \quad (+2)$$
$$+ e^{-3(t-2\pi)}\sin 4(t-2\pi)\,u(t-2\pi) \quad (+2)$$

$$= \begin{cases} e^{-3t}\cos 4t + e^{-3t}\sin 4t & \text{if } 0 < t < \pi \\ e^{-3t}\cos 4t + (1+e^{3\pi})\,e^{-3t}\sin 4t & \text{if } \pi < t < 2\pi \\ e^{-3t}\cos 4t + (1+e^{3\pi}+e^{6\pi})\,e^{-3t}\sin 4t & \text{if } t > 2\pi \end{cases}$$