

Quiz #1 (CSE 400.001)

Thursday, March 11, 2004

Name: _____ E-mail: _____

Dept: _____ ID No: _____

1. (10 points) Solve the following equation:

$$\underbrace{(3xy + y^2)}_P + \underbrace{(x^2 + xy)}_Q y' = 0$$

$$\frac{1}{H} \frac{dH}{dx} = \frac{1}{Q} (P_y - Q_x) = \frac{1}{x^2 + xy} (3x + 2y - 2x - y)$$

$$= \frac{1}{x} \quad (+2)$$

$$H(x) = \exp(\ln x) = x \quad (+2)$$

$$(3x^2y + xy^2)dx + (x^3 + x^2y)dy = 0 \quad (+2)$$

$$u(x, y) = \int (3x^2y + xy^2) dx$$

$$= x^3y + \frac{1}{2}x^2y^2 + h(y) \quad (+2)$$

$$\frac{\partial u}{\partial y} = x^3 + x^2y + h'(y) = x^3 + x^2y \quad (+1)$$

$$\therefore h(y) = \hat{c}$$

$$\therefore u(x, y) = x^3y + \frac{1}{2}x^2y^2 = C \quad (+1)$$

2. (10 points) Reduce to a linear form and solve the following equation. Show all the steps of your work.

$$2xyy' + (x-1)y^2 = x^2e^x.$$

$$v = y^2, \quad v' = 2yy' \quad (+3)$$

$$xv' + (x-1)v = x^2e^x \quad (+2)$$

$$v' + (1 - \frac{1}{x})v = xe^x \quad (+1)$$

$$\begin{aligned} v &= e^{-\int(1-\frac{1}{x})dx} \left[\int e^{\int(1-\frac{1}{x})dx} xe^x dx + c \right] \\ &= e^{-x} \cdot x \left[\int e^x \cdot \frac{1}{x} \cdot x \cdot e^x dx + c \right] \\ &= e^{-x} \cdot x \left[\frac{1}{2} e^{2x} + c \right] \\ &= \frac{1}{2} x e^x + c x e^{-x} \end{aligned} \quad (+3)$$

$$y = \pm \sqrt{v} = \pm \sqrt{\frac{1}{2} x e^x + c x e^{-x}} \quad (+1)$$