Quiz #3 (CSE 400.001)

Thursday, April 8, 2004

Name: ______________________ **E-mail:** ______________________

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1. (10 points) Derive the following formula, showing the details of your work

$$\mathcal{L}^{-1}\left[\frac{s}{s^4+4a^4}\right] = \frac{1}{2a^2}\sinh at\ \sin at$$

$$\sinh at \cdot \sin at = \frac{1}{2}(e^{at} - e^{-at}) \sin at \quad +3$$

$$\mathcal{L}[\sinh at \cdot \sin at]$$

$$= \frac{1}{2}\left[\frac{a}{(s-a)^2+a^2} - \frac{a}{(s+a)^2+a^2}\right]$$

$$= \frac{1}{2}\left[\frac{a}{s^2+2a^2-2as} - \frac{a}{s^2+2a^2+2as}\right] \quad +3$$

$$= \frac{1}{2} \cdot \frac{4a^2 s}{(s^2+2a^2)^2 - 4a^2 s^2}$$

$$= \frac{2a^2 s}{s^4+4a^4} \quad +2$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^4+4a^4}\right] = \frac{1}{2a^2} \cdot \sinh at \cdot \sin at \quad +2$$

2. (15 points) Using the Convolution theorem, find the following inverse transform

$$\mathcal{L}^{-1}\left[\frac{2s+6}{(s^2+6s+10)^2}\right]$$

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$$= \mathcal{L}^{-1}\left[2\cdot\frac{s+3}{(s+3)^2+1}\cdot\frac{1}{(s+3)^2+1}\right] \quad +3$$

$$= 2\cdot\left(e^{-3t}\cos t\right) * \left(e^{-3t}\sin t\right) \quad +4$$

$$= 2\int_0^t e^{-3\tau}\cos\tau\cdot e^{-3(t-\tau)}\sin(t-\tau)\,d\tau$$

$$= 2e^{-3t}\int_0^t \cos\tau\cdot\sin(t-\tau)\,d\tau \quad +3$$

$$= 2e^{-3t}\cdot\frac{1}{2}\left[\int_0^t \sin t\,d\tau - \int_0^t \sin(2\tau-t)\,d\tau\right] \quad +2$$

$$= e^{-3t}\left[t\sin t + \frac{1}{2}\underbrace{\left[\cos(2\tau-t)\right]_0^t}_{=0}\right] \quad +2$$

$$= e^{-3t}\,t\sin t \quad +1$$