Quiz #2 (CSE 400.001)

Monday, September24, 2012

Name: _____ **E-mail:** _____

Dept: _____ **ID No:** _____

1. (8 points) Solve the following differential equation:

$$x^2y'' + xy' + y = 1 + x + x^2, \quad x > 0.$$

$m(m-1) + m + 1 = 0$

$m^2 + 1 = 0$

$m = \pm i$ (+2)

$y_h = A\cos(\ln x) + B\sin(\ln x)$ (+1)

$y_p = c_1 + c_2 x + c_3 x^2$ (+2)

$y_p' = c_2 + 2c_3 x$

$y_p'' = 2c_3$

$x^2y_p'' + xy_p' + y_p = c_1 + 2c_2 x + 5c_3 x^2$

$= 1 + x + x^2$ (+2)

$\therefore c_1 = 1, \; c_2 = \frac{1}{2}, \; c_3 = \frac{1}{5}$

$y = y_h + y_p = A\cos(\ln x) + B\sin(\ln x)$

$+ 1 + \frac{1}{2}x + \frac{1}{5}x^2$ (+1)

2. (12 points) Solve the following initial value problem:

$$y''' - 3y'' + 3y' - y = x^2e^x, \quad y(0) = 1,\ y'(0) = 2,\ y''(0) = 3.$$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = (\lambda-1)^3 = 0$

$y_c = (c_1 + c_2x + c_3x^2)e^x$

$y_1 = e^x,\ y_2 = xe^x,\ y_3 = x^2e^x$] +3

$W = 2e^{3x},\ W_1 = x^2e^{2x},$

$W_2 = -2xe^{2x},\ W_3 = e^{2x}$] +2

$y_p = \sum y_i \int \frac{W_i}{W} r(x)dx = \frac{1}{60}x^5e^x$ +3

$\therefore y = y_c + y_p = (c_1 + c_2x + c_3x^2)e^x + \frac{1}{60}x^5e^x$ +1

$y' = [(c_1+c_2) + (c_2+2c_3)x + c_3x^2]e^x + \cdots$

$y'' = (c_1 + 2c_2 + 2c_3)e^x + \cdots$] +2

$\therefore c_1 = 1,\ c_2 = 1,\ c_3 = 0$

$\therefore y = e^x + xe^x + \frac{1}{60}x^5e^x$ +1