Quiz #6 (CSE 400.001)

NDecember 3, 2012 (Monday)

Name: ______________________ **ID No:** ______________________

1. (10 points)

   (a) (5 points) If a square matrix A has column 1 + column 2 = column 3, show that A is not invertible.

   (b) (5 points) If a square matrix A has a row of zeros, show that A is not invertible.

(a)

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}$$

Let $x = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, then $Ax = a_1 + a_2 - a_3 = 0$.

But, $x \neq 0$.

$\therefore$ A is not invertible

(b) Suppose A has an inverse $A^{-1} = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$.

A has a row of zeros (say, the ith row)

$\Rightarrow$ The ith element of Ax_i should be 0.

But, it is impossible since $Ax_i = e_i$.

$\therefore$ A is not invertible

2. (10 points) Solve $A\mathbf{x} = \mathbf{b}$ using the triangular systems $L\mathbf{c} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{c}$.

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

What part of A^{-1} have you found with this particular $\mathbf{b}$?

(a) $$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow c_1 = 0, \ c_2 = 0, \ c_3 = 1$

(b) $$\begin{bmatrix} 2 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow x_3 = 1, \ x_2 = -3, \ x_1 = 1$

(c) $\mathbf{b} = \mathbf{e}_3 \Rightarrow \mathbf{x} = A^{-1}\mathbf{b} = A^{-1}\mathbf{e}_3$

$\therefore$ $\mathbf{x}$ is the third column of A^{-1}