Quiz #5 (CSE 400.001)

Monday, December 2, 2013

Name: ______________________ **ID No:** ______________________

1. (8 points) Suppose A is the 4 by 4 identity matrix except for a vector $\mathbf{v}$ in column 2:

$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}.$$

Factor A into LU, assumung $v_2 \neq 0$, and find A^{-1}.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & v_3/v_2 & 1 & 0 \\ 0 & v_4/v_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = LU$$

+2 +1

$$A^{-1} = U^{-1} L^{-1}$$

$$= \begin{bmatrix} 1 & -v_1/v_2 & 0 & 0 \\ 0 & 1/v_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -v_3/v_2 & 1 & 0 \\ 0 & -v_4/v_2 & 0 & 1 \end{bmatrix}$$

+2 +2

$$= \begin{bmatrix} 1 & -v_1/v_2 & 0 & 0 \\ 0 & 1/v_2 & 0 & 0 \\ 0 & -v_3/v_2 & 1 & 0 \\ 0 & -v_4/v_2 & 0 & 1 \end{bmatrix}$$ +1

2. (10 points) True or false, with reason if true or counterexample if false:

(a) (3 points) If A is invertible and its rows are in reverse order in B, then B is invertible.

(b) (2 points) If A and B are symmetric, then AB is symmetric.

(c) (2 points) If A and B are invertible, then BA is invertible.

(d) (3 points) Every nonsingular matrix can be factored into the product $A = LU$ of a lower triangular L and an upper triangular U.

(a) True

Assume B is not invertible, then

$Bx = 0$ for some $x \neq 0$.

$Ax = PBx = P \cdot 0 = 0$, where $P = \begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix}$.

Thus, A is not invertible #

(b) False.

Counterexample:

$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$: symmetric.

But, $AB = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ is not symmetric

(c) True

$(BA)(A^{-1}B^{-1}) = B(AA^{-1})B^{-1} = BB^{-1} = I$

$(A^{-1}B^{-1})(BA) = A^{-1}(B^{-1}B)A = A^{-1}A = I$

$\therefore$ BA is invertible and $(BA)^{-1} = A^{-1}B^{-1}$

(d) False

Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Assume $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ l_{21}u_{11} & * \end{bmatrix}$

Then, $u_{11} = 0$, and $1 = l_{21}u_{11} = 0$ #

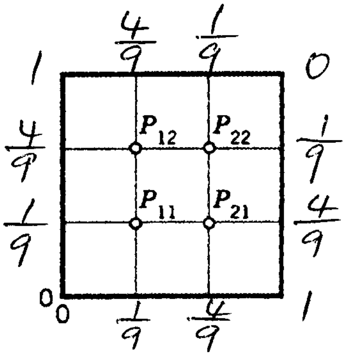
3. (12 points) Using $h = k = 1/3$, approximate the solution to the following elliptic equation

$$u_{xx} + 4u_{yy} = 9, \quad 0 < x < 1,\ 0 < y < 1$$

with boundary conditions:

$$u(x,0) = x^2, \quad u(x,1) = (x-1)^2, \quad 0 \le x \le 1;$$
$$u(0,y) = y^2, \quad u(1,y) = (y-1)^2, \quad 0 \le y \le 1.$$

Set up a system of linear equations.



i	j	x_i	y_j	$u(x_i, y_j)$	i	j	x_i	y_j	$u(x_i, y_j)$
1	1	1/3	1/3		2	1	2/3	1/3	
1	2	1/3	2/3		2	2	2/3	2/3	

$$u_{xx} + 4u_{yy} = 9$$

$$\frac{1}{h^2}\left(u_{i+1,j} - 2u_{ij} + u_{i-1,j}\right) + \frac{4}{k^2}\left(u_{i,j+1} - 2u_{ij} + u_{i,j-1}\right) = 9 \quad +2$$

$$u_{i-1,j} + 4u_{i,j-1} - 10u_{ij} + 4u_{i,j+1} + u_{i+1,j} = 1 \quad +2$$

at P_{11}: $\underbrace{\cancel{u_{01}}}_{1/9} + 4\underbrace{\cancel{u_{10}}}_{4/9} - 10u_{11} + 4u_{12} + u_{21} = 1$

$$-10u_{11} + 4u_{12} + u_{21} = 4/9 \quad +2$$

at P_{12}: $4u_{11} - 10u_{12} + u_{22} = -11/9 \quad +2$

at P_{21}: $u_{11} - 10u_{21} + 4u_{22} = -11/9 \quad +2$

at P_{22}: $u_{12} + 4u_{21} - 10u_{22} = 4/9 \quad +2$