Name: _____   Dept: _____   ID No: _____

1. (10 points) A transformation from R^1 to R^1 sends 1 to 1; 2 to 2; and 3 to 7.

   (a) (5 points) What is the matrix representation of this transformation?

   (b) (3 points) This transformation sends t to $f(t)$. What is the representation of $f(t)$ as a linear rational function of t?

   (c) (2 points) Which value of t goes to infinity under this transformation?

(a) $\begin{bmatrix} A & B \\ C & 1 \end{bmatrix} \begin{bmatrix} t_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_i \\ 1 \end{bmatrix}$, $t = 1, 2, 3$

$A t_i + B = f_i (C t_i + 1)$

$\begin{cases} A + B - C = 1 \\ 2A + B - 4C = 2 \\ 3A + B - 7C = 7 \end{cases}$ $\Rightarrow A - 3C = 1$

$\Rightarrow A - 17 C = 5$

$\therefore A = \frac{1}{7}, \quad C = -\frac{2}{7}, \quad B = \frac{4}{7}$

$\begin{bmatrix} 1 & 4 \\ -2 & 7 \end{bmatrix}$

(b) $f(t) = \dfrac{t + 4}{-2t + 7}$

(c) $t = \dfrac{7}{2}$